Yuval Peres, Microsoft Research, One Microsoft Way WA, 98052, U.S.A.

HAUSDORFF DIMENSION FOR SUBSHIFTS INVARIANT UNDER THE MULTIPLICATIVE INTEGERS.

Consider the "multiplicative golden shift", i.e., the set $M(G)$ of real numbers x in the unit interval, such that for all n, the nth and $2n$th binary digits of x are not both 1. We find explicitly the Hausdorff dimension of $M(G)$, and show that it is strictly smaller than the Minkowski dimension. We put this in a general context of invariance under the semigroup of multiplicative integers and relate $M(G)$ to self-affine sets. The question we solve was raised by Fan, Liao, and Ma (2010) who pointed out a connection to multifractal analysis for nonconventional ergodic averages. Many open problems remain in this topic, and I will emphasize them. (Joint work with Rick Kenyon and Boris Solomyak).