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TOPOLOGICAL HAUSDORFF DIMENSION AND LEVEL SETS ON FRACTALS

We introduce a new concept of dimension for metric spaces, the so called *topological Hausdorff dimension*. Its definition is a very natural combination of the definitions of the topological dimension and the Hausdorff dimension. The value of the topological Hausdorff dimension is always between the topological dimension and the Hausdorff dimension, in particular, this new dimension is a non-trivial lower estimate for the Hausdorff dimension.

We examine the basic properties of this new notion of dimension, compare it to other well-known notions, determine its value for some classical fractals such as the Sierpiński carpet, the von Koch snowflake curve, Kakeya sets, trail of the Brownian motion, etc.

As our first application, we generalize the celebrated result of Chayes, Chayes and Durrett about the phase transition of the connectedness of the limit set of Mandelbrot's fractal percolation process. They proved that certain curves show up in the limit set when passing a critical probability, and we prove that actually 'thick' families of curves show up, where roughly speaking the word thick means that the curves can be parametrized in a natural way by a set of large Hausdorff dimension. The proof of this is basically an estimate of the topological Hausdorff dimension of the limit set from below. For the

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sake of completeness, we also give an upper estimate and conclude that in the non-trivial cases the topological Hausdorff dimension is almost surely strictly below the Hausdorff dimension.

Finally, as our second application, we show that the topological Hausdorff dimension is precisely the right notion to describe the Hausdorff dimension of the level sets of the generic continuous function (in the sense of Baire category) defined on a compact metric space.