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ON IDEALS WHICH COULD BE ASSOCIATED TO A POSET OF TREES

Let $(\mathbb{Q}, \leq)$ be a poset. Consider the following types of ideals, which are associated to $(\mathbb{Q}, \leq)$.

- $(q^0) = \{S \subseteq \bigcup \mathbb{Q} : \forall_{p \in \mathbb{Q}} \exists_{q \leq p} \;\; q \cap S = \emptyset\}$;
- $(q^1) = \{S \subseteq \mathbb{Q} : \forall_{p \in \mathbb{Q}} \exists_{q \leq p} \;\; \{y : y \leq q\} \cap S = \emptyset\}$;
- $(q^2) = \{S \subseteq \mathbb{Q}^* : \forall_{p \in \mathbb{Q}} \exists_{q \leq p}) \;\; q^* \cap S = \emptyset\}$.

If Q is a collection of trees, e.g. an arboreal forcing condition like in [3], then meaning of $\bigcup \mathbb{Q}$ is cleared. Formally, trees are contained in Seq_X (finite sequences of elements from X) and any tree $\mathcal{T} \subseteq Seq_X$ one can identify with the set $[\mathcal{T}] \subseteq X^\omega$ of all its infinite branches. $\mathbb{Q}^*$ denotes the collection of all maximal centered families which are contained in $\mathbb{Q}$ and $p^* = \{U \in \mathbb{Q}^* : p \in U\}$.

Results about:

- The ideal (q^2) for $([\omega]^\omega, \subseteq^*)$, see [1] and [2];
- (q^0) for $([\omega]^\omega, \subseteq^*)$ or the Mathias forcing conditions, compare [8] and [9];
- (q^0) for the Silver forcing or n-Silvers forcing conditions, compare [6] and [7];

are generalized for system of of trees.

System of (Laver) trees where examined in [5] and [4]. Given

$$< A_s \in [\omega]^\omega : s \in \omega^{<\omega} >= \overline{A}$$

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define $p_s(\overline{A})$ to be the unique Laver tree such that the root of $p_s(\overline{A})$ is s and for every node $t \supseteq s$ with $t \in p_s(\overline{A})$ we have that

$$split(p_s(\overline{A}), t) = A_t.$$

Define $\overline{A} \subseteq^* \overline{B}$ iff $A_s \setminus B_s$ is finite for all $s \in \omega^{<\omega}$, see [5], and define $\overline{A} \prec^* \overline{B}$ iff $A_s \subseteq B_s$ for all but finite many $s \in \omega^{<\omega}$, see [4]. The poset $\subseteq^*$ is separative, so one can directly adopt Base Matrix Tree Theorem and Kulpa - Szymański Theorem similarly as in [1], [2]. The poset $\prec^*$ is not separative, so it need a non separative version of Base Matrix Tree Theorem. Counter-examples of systems of trees without properties needed for non separative version of the Base Trees Theorem are given, too.

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