Jack Grahl , Maths Dept. UCL Gower Street London WC1E 6BT United Kingdom.   email: `jgrahl@math.ucl.ac.uk`

STRONG CONVERGENCE OF THE RANDOM RIEMANN INTEGRAL

The random Riemann sum is a Riemann sum where the tag point in each interval of a partition is a random variable distributed uniformly w.r.t. Lebesgue measure and independently of each other tag point. For a given function and sequence of partitions we can ask if the corresponding sequence of random Riemann sums converges almost surely. Previously a sufficient condition for this was given, namely that the function be in L^p and the sequence of partition sizes be in l^{p-1} for some p. We give a partial converse to this, namely that for a restricted class of functions this is also a necessary condition. We also construct a counterexample which shows why the condition can not be necessary for all measurable functions.