Real Analysis Exchange
Summer Symposium 2011, pp. 46–47

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DISTANCES TO SPACES OF FIRST H-CLASS FUNCTIONS

Using a notion of first H-class function given by G. Koumoullis in [4] and σ-fragmentability given by J.E. Jayne et al. in [2] and [3] we generalize results about distances to spaces of Baire one functions presented in [1].

Further, we introduce new concepts of fragmentability and investigate their properties.

Given a topological space X and a metric space E we use index of σ-fragmentability by resolvable sets for maps $f : X \to E$ to estimate the distance of f to the space of first H-class functions (i.e., functions $g : X \to E$ such that $g^{-1}(U)$ can be expressed as a countable union of resolvable sets in X for every open set U in E).

The notion of $H_1(X, E)$ spaces originates from [4] and our goal is to state analogous results for this spaces as was given in [1] for the spaces of Baire one functions. We prove that under certain assumptions (a space X is topologically Hansell or space E is separable) holds

$$\frac{1}{2}\operatorname{frag}_r(f) \leq d(f, H_1(X, E)) \leq \operatorname{frag}_r(f).$$

As a corollary we get that provided X is a hereditarily Baire space holds

$$\frac{1}{2}\operatorname{frag}(f) = d(f, H_1(X, \mathbb{R})).$$

Further, some other indices of fragmentability are developed and their properties are examined.

Mathematical Reviews subject classification: Primary: 26A21; Secondary: 54C30, 54.60
Key words: fragmented functions, oscillation, distances
*The research was supported by GAČR 401/09/H007 and SVV-2011-26331.

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