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COVERING AN ADDITIVE FUNCTION BY LESS THAN CONTINUUM MANY CONTINUOUS FUNCTIONS

A function $f\colon \mathbb{R} \to \mathbb{R}$ is:

- *countably linear*, if f can be covered by countably many linear functions (i.e. functions $f : \mathbb{R} \to \mathbb{R}$ of the form $f(x) = ax+b$, where a, b are fixed);
- *strongly countably continuous*, if there is a sequence $\langle f_n \rangle_{n<\omega}$ of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ such that $f \subset \bigcup_{n<\omega} f_n$.
- *strongly $<\mathfrak{c}$-continuous* if there is a cardinal $\kappa < \mathfrak{c}$ and a sequence $\langle f_\alpha \rangle_{\alpha<\kappa}$ of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ such that $f \subset \bigcup_{\alpha<\kappa} f_\alpha$.

We prove that if a function f is additive and strongly countably continuous then it is countably linear [2]. Thus CH implies that every additive strongly $<\mathfrak{c}$-continuous function can be covered by less than $\mathfrak{c}$ many linear functions. We show that the negation of this statement is consistent with ZFC. This is a consequence of the existence of a "nice" Hamel basis which is a union of less than continuum many perfect sets [3]. It is known that such bases exist in the iterated perfect set model [1]. Moreover, we prove in ZFC that each additive function which can be covered by less than continuum many lines passing through the point $\langle 0, 0\rangle$ is continuous.

References

[1] K. Ciesielski, J. Pawlikowski, *Nice Hamel bases under the Covering Property Axiom*, Acta Math. Hungar. 105(3), (2004), 197–213.

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[2] T. Natkaniec, *On additive countably continuous functions*, Publ. Math. Debrecen, to appear.

[3] T. Natkaniec, *Covering an additive function by* $< \mathfrak{c}$ *many continuous functions*, in preparation.