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INFINITE GAMES AND σ-POROSITY

We characterize σ-P-porous sets in a compact metric space via an infinite game where P is a porosity-like relation satisfying some additional conditions. This can be applied to ordinary porosity above all but also to some other variants of porosity. We use the game above to prove that there exists a compact and non-σ-porous subset of a given Borel and non-σ-porous set in any locally compact metric space and also analogous results for symmetrical and strong porosity (the last one is a new result). Further, we show that there exists a closed set which is σ-$(1-\varepsilon)$-symmetrically porous for every $0 < \varepsilon < 1$ but which is not σ-1-symmetrically porous.

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