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LINEARIZABLE SYSTEMS OF TWO SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS

Linear systems of ordinary differential equations (ODEs) are applicable in analysis. But in applications they often occur in disguised forms. It is only after a change of variables that the underlying linear structure of a nonlinear system is uncovered. We consider systems of two second-order ODEs. Criteria of their linearization by a point transformation to the simplest form x"=0, y"=0 are well known. Also it is known that arbitrary linear system may be irreducible by a point transformation to this simplest system. Hence, there exist linearizable systems, which cannot be mapped to the simplest form by an invertible change of variables. We provide the criteria of reducibility of a nonlinear system by a point transformation to the arbitrary linear system. These criteria are applied to n-body problem (when n=2), geodesic equations and some other systems. Once a given system is linearized then the next problem is to integrate the linear system obtained. The solution is obvious when it has the form x"=0, y"=0. We consider the group of equivalence transformations of linear systems of general form and solve the equivalence problem for this type of equations. It turns out that all linear systems fall into five classes. The most degenerate class consists of all linear systems reducible by a suitable change of variables to the simplest form. For the remaining four classes of linear systems we provide the basis of differential invariants of the group of equivalence transformations. Using the invariants one can easily ascertain if a given linear system is reducible to the form simple for integration (for example, to the system with a separating equation, decoupled system, system with constant coefficients and so on). Some examples of applying the invariants of linear systems are considered.