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ON SUBSERIES OF DOUBLE SERIES

Abstract

Attempt has been made in this paper to investigate some properties of subseries of an infinite double series using the notion of asymptotic density of the original series and also some set theoretic properties of the sets $C(\sum\sum a_{mn})$ and $D(\sum\sum a_{mn})$, defined in the paper have been discussed.

1 Introduction.

Investigation on subseries of an infinite series of real terms have been found prominent position in the litarature during last several decades.The authors [4],[5] proved several interesting and deep theorems on double series for different rearrangement and techniques. Study has been mainly concentrated on subseries of a double series of non negative terms using the concept of asymptotic density in the original series.
We write a double series $\sum\sum a_{mn}$ in full length as follows:

$$a_{11} + (a_{21} + a_{22} + a_{12}) + (a_{31} + a_{32} + a_{33} + a_{23} + a_{13}) + ... + (a_{n1} + a_{n2} + ... + a_{nn} + a_{(n-1)n} + a_{(n-2)n} + ... + a_{2n} + a_{1n}).........(1)$$

Let $x \in (0, 1]$.Then x can be expressed as

$$x = \frac{x_1}{2}+\frac{x_2}{2^2}+\frac{x_3}{2^3}+....,$$

where $x_i \in \{0, 1\}$ for all i and $x_i = 1$ for infinitely many i.

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Let A=$\{n : x_n(x) = 1\}$. We define a sub series $\sum\sum \varepsilon_{mn}(x)a_{mn}$ as follows: We take those term of $\sum\sum a_{mn}$ if the integers corresponding to the position of the term of the double series in (1) are present in A,
otherwise we omit them.Thus to each $x \in (0, 1]$, there correspond a subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$,.......(2)
where $\varepsilon_{mn}(x) = 1$, if the position of the term a_{mn} in (1) is a member of A , otherwise we set $\varepsilon_{mn}(x) = 0$.
Therefore to each $x \in (0.1]$, we can associate a subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$ of the given double series $\sum\sum a_{mn}$ and conversely.

Let $p(m, n, x) = \sum_{\alpha \in A, \alpha \leq mn} 1$,
The numbers

$$p_1(x) = \liminf \frac{p(m,n,x)}{mn},$$
$$p_2(x) = \limsup \frac{p(m,n,x)}{mn}$$

are called the lower and upper asymptotic density of the series (2).
If $p(x) = \lim \frac{p(m,n,x)}{mn}$ exist then $p(x)$ is called asymptotic density of the series (2) with respect to the series (1).
We shall state two theorems on subseries of double series with nonnegative terms.

2 Main Results.

Theorem 1. *Let $\{a_{mn}\}$ be a decreasing sequence of positive terms converging to 0 and $\liminf mna_{mn} > 0$. If for some $x \in (0, 1]$, the corresponding subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$ of the double series $\sum\sum a_{mn}$ be convergent, then x satiesfies the condition $p(x) = \lim \frac{p(m,n,x)}{mn} = 0$.*

Theorem 2. *Let $\sum\sum a_{mn}$ be a divergent series of positive terms and let there exist a positive integer μ such that $a_{mn} \leq a_{pq}$ whenever $m \geq p \geq \mu, n \geq q \geq \mu$. If for some $x \in (0, 1]$,the corresponding subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$ of the double series $\sum\sum a_{mn}$ is convergent, then x satiesfies the condition $p_1(x) = \liminf \frac{p(m,n,x)}{mn} = 0$.*

3 Topological properties on subseries of double Series.

Let $\sum\sum a_{mn}$ denote a double series with positive real numbers. The set of all those $x \in (0.1]$ represented by a dyadic scale $x = \sum_{k=1}^{\infty} \varepsilon_k(x)2^{-k}, \varepsilon_k(x) = 0$ or 1 for all $k \in N$, the set of natural numbers but $\varepsilon_k(x) = 1$ for infinitely many k, for which the subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$ is convergent (divergent) will be denoted by $C(\sum\sum a_{mn})(\ D(\sum\sum a_{mn}))$. Let $G(\sum\sum a_{mn})$ denote the set of all $x \in (0,1]$ for which the subseries $\sum\sum \varepsilon_{mn}(x)a_{mn}$ has bounded partial sum. In this section the properties of the sets $C(\sum\sum a_{mn})$ and $D(\sum\sum a_{mn})$ will be studied.

Theorem 3. *The set* $C(\sum\sum a_{mn})$ *is of first category and also* G_δ *set ,where* $C(\sum\sum a_{mn})$ *is the set of all points in* $(0,1]$ *for which the subseries* $\sum\sum \varepsilon_{mn}(x)a_{mn}$ *is convergent.*

Corollary 4. *The set* $D(\sum\sum a_{mn})$ *is a* G_δ *residual set,where* $D(\sum\sum a_{mn})$ *is the set of all points in* $(0,1]$ *for which the subseries* $\sum\sum \varepsilon_{mn}(x)a_{mn}$ *is divergent.*

Theorem 5. *Let* $\sum\sum |a_{mn}| = +\infty$ *and* 0 *be a limit point of the sequence* $\{a_{mn}\}$*. Then*
$[C(\sum\sum a_{mn})]^c = [D(\sum\sum a_{mn})]^c = [0,1]$.
[*the symbol* A^c *stands for condensation point of the set* A]

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