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STACKING SQUARES

Let $H = \bigcup_i H_i$ be the union of finitely many unit squares in the plane. Is there an upper bound on the ratio of H's perimeter to H's area? This question was first posed by Tamás Keleti in 1998 on the famous Hungarian Schweitzer competition [7]. Keleti conjectured that this upper bound is exactly 4. In [5], Keleti's student Zoltan Gyenes proved an upper bound of 5.6 and proved the following theorem.

Theorem 1. *If the H_i are assumed to be axis oriented, then the ratio of H's perimeter and area cannot exceed 4.*

In the same paper, Gyenes showed that many natural generalizations of Keleti's paper fail. Notably, the corresponding ratio for unions of congruent convex sets need not be bounded by the ratio for a single copy of the set.

We give an original inductive proof of Theorem 1 showing that when a square is added the ratio of the change in perimeter and change in area is less than 4. However, this method fails to generalize; if multiple orientations are allowed the ratio of change in perimeter and change in area may be arbitrarily large.

Finally, we examine the nature of a potential counterexample to Keleti's conjecture. Using the isoparametric inequality, we show that in a counterexample with a minimal amount of squares, the squares overlap to a large degree.

Theorem 2. *If H is a counterexample with a minimal number of squares, then the area of $H_i \cap (H \setminus H_i)$ is strictly greater than $\frac{\pi}{4}$ for each i.*

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In a forthcoming paper, we explore how the perimeter area ratio is affected when shifting or rotating H's component squares.

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