

Aliasghar Alikhani-Koopaei, Department of Mathematics
Penn State University, Berks Campus, Berk, PA 19610.
email: axa12@psu.edu

ON THE SET OF PERIODIC POINTS OF CONTINUOUS AND CONTINUOUSLY DIFFERENTIABLE SELF-MAPS OF INTERVALS

Motivated by common fixed point conjecture, more than a decade ago, we asked if two commuting continuous self-maps of an interval do share a common periodic point. This problem still remains open, however it has led us to study the size of the set of periodic points and some other sets related to the dynamics of self-maps of an interval. It has become clear that when the set of periodic points of one of the two commuting continuous functions is small or large in some sense, the two functions will more likely share a periodic point. For example the countability of the periodic point set of one of the functions will grantee that they have a common periodic point. A. J. Schwartz in mid sixties showed that if f and g commute and f has the extra condition of being continuously differentiable, then f and g must have a common fixed point. This result along with our observation may suggest that some other class of functions including the class of continuously differentiable functions must have a countable set of periodic points. In this note we give a brief history of the common periodic point conjecture, state some related open questions, and present some new results related to the typical behavior of continuously differentiable functions.

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