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LINEARLY CONTINUOUS FUNCTIONS: REGULARITY AND GENERALIZATION

A function $f : \mathbb{R}^n \to \mathbb{R}$ is linearly continuous if for every line $L \subset \mathbb{R}^n$, the restriction $f|L$ of f to L is continuous. It has been known for some time that there are linearly continuous functions which are discontinuous. In this talk, we discuss some results of the authors' which describe how discontinuous these functions can be. We present descriptive structural results for the sets of discontinuity, and a partial characterization of these sets, as well as some basic results on the Baire class on these functions. We discuss several related classes of real functions on $\mathbb{R}^n$ and similar results which hold for them as well.

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