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SEPARABLE REDUCTION THEOREMS BY THE METHOD OF ELEMENTARY SUBMODELS

We simplify the presentation of the method of elementary submodels and we show that it can be used for simplifying proofs of existing separable reduction theorems and for obtaining new ones (for details about the method see [1]). Given a nonseparable Banach space X and either a subset $A \subset X$ or a function f defined on X, we are able for certain properties produce a separable subspace of X which determines whether A or f has the property. Such results are proved for properties of sets "to be dense, nowhere dense, meager, residual, porous or σ-porous" and for properties of functions "to be continuous, semicontinuous or Fréchet differentiable". Our method of creating separable subspaces enables us to combine our results, so we easily get separable reductions of function properties such as "be continuous on a dense subset", "be Fréchet differentiable on a residual subset", etc. Finally, we show some applications of presented separable reduction theorems and demonstrate that some results of Zajíček (see [3]), Lindenstrauss and Preiss (see [4]) hold in nonseparable setting as well. All the results are contained in [1] except the result about σ-porosity which may be found in [2].

References

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