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TRANSFERENCE OF DENSITY MAY BE POSSIBLE

Sierpinski's classical example of a set in the plane containing all but countably many points on each vertical line, but only finitely many points on each horizontal line shows that transference of density may not be possible. However, Sierpinski's construction uses the continuum hypothesis. In an earlier work Laczkovich and I showed that the possibility of such transference organically depends on the underlying set-theoretic axioms. In this work we consider what can be said in the broader context of multiple directions.

We consider subsets of the upper half-plane, and investigate the possibility of transferring one variety of linear density at points of the real line to another variety of linear density. We give a complete solution to this problem in that we try to find the optimal results whether the sets in question are measurable or not. In the nonmeasurable case the best possible results prove to be independent of ZFC, and depend on the nonexistence of certain sets with paradoxical properties.

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Mathematical Reviews subject classification: Primary: 03E35; Secondary: 28A20, 28A03

Key words: density of sets, weak Sierpinski sets

*This research was done in collaboration with M. Laczkovich

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