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## DISTRIBUTIONAL CHAOS AFTER 16 YEARS

In 1994 B. Schweizer and J. Smítal published the paper on distributional chaos for continuous maps of the interval. This notion was later generalized, and currently there are many versions of this type of chaos, suitable for continuous maps of a compact metric space. More than 300 papers were already written on the subject. In the talk we point out main recent results, main open problems and possible directions of further research.

To recall the basic definitions, let  $(X, \rho)$  be a compact metric space, and  $f \in C(X)$  a continuous map  $f : X \rightarrow X$ . For  $x, y \in X$  and  $n > 0$ , define the *lower* and *upper distribution functions*  $\Phi_{xy}, \Phi_{xy}^* : (0, \infty) \rightarrow I$  by

$$\Phi_{xy}(t) = \liminf_{n \rightarrow \infty} \frac{1}{n} \#\{0 \leq i < n; \rho(f^i(x), f^i(y)) < t\},$$
$$\Phi_{xy}^*(t) = \limsup_{n \rightarrow \infty} \frac{1}{n} \#\{0 \leq i < n; \rho(f^i(x), f^i(y)) < t\}.$$

Then  $\Phi_{xy}(t) \leq \Phi_{xy}^*(t)$  are non-decreasing functions, and  $\Phi_{xy}(t) = 1$  if  $t > \text{diam}(X)$ . Let  $\Phi_{xy} < \Phi_{xy}^*$  mean that  $\Phi_{xy}(t) < \Phi_{xy}^*(t)$  for all  $t$  in an interval. The three versions of *distributional chaos*, briefly *DC1*, *DC2*, or *DC3*, appear when there are  $x, y \in X$  such that

(DC1)  $\Phi_{xy}^* \equiv 1$  and  $\Phi_{xy}(t) = 0$ , for some  $t > 0$

(DC2)  $\Phi_{xy}^* \equiv 1$  and  $\Phi_{xy} < \Phi_{xy}^*$

(DC3)  $\Phi_{xy}^* > \Phi_{xy}$

Recall that *DC1* is the original notion introduced by *Schweizer & Smítal* (1994), *DC2* and *DC3* are generalizations by *Štefánková & Smítal* (2004), and *Balibrea, Štefánková & Smítal* (2005), respectively.

Below is a list of authors of main contributions to the theory of distributional chaos.

**1994**

B. Schweizer and J. Smítal

**1998**

G. L. Forti and L. Paganoni

G. F. Liao and Q. J. Fan

D. G. Ma

M. Málek

**1999**

M. Babilonová

F. Z. Du, L. D. Wang, Y. Y. Gai

G.-L. Forti, L. Paganoni and J. Smítal

G. F. Liao and Q. I. Fan

M. Málek

L. Wang and G. Liao

**2000**

M. Babilonová

G. F. Liao and Q. J. Fan

A. Sklar and J. Smítal

X. D. Xie

**2001**

J. S. Cánovas

J. S. Cánovas, L. G. Soler and M. M. Ruiz

T. X. Guo

K. Janková

M. J. Li and K. T. Li

R. Pikula

S. J. Zhan

**2002**

J. Bobok

G. F. Liao, and L. J. Wang

G. F. Liao, L. D. Wang

Z. M. Luo, K. C. Zhang

B. Schweizer, A. Sklar and J. Smítal

W. Song, L. J. Wang, A. H. Zhang

R. S. Yang

L. D. Wang, G. F. Liao, Y. Yang

**2003**

M. Babilonová

F. Balibrea, B. Schweizer, A. Sklar and J. Smítal

L. Wang, G. Liao, Z. Chen, and X. Duan

**2004**

J. S. Cánovas and R. Hric

J. L. Garcia Guirao

M. Málek

J. Smítal and M. Štefánková

W. Song, G. F. Liao, and B. H. He

L. Wang, G. Liao, Z. Chu, and X. Duan

L. Wang, Z. Chu and G. Liao

**2005**

G.-L. Forti

F. Balibrea, J. Smítal and M. Štefánková

L. Paganoni and J. Smítal

H. Y. Wang and J. C. Xiong

**2006**

J. S. Canovas, A. Linero and D. Peralta-Salas

E. D'Aniello

J. L. Garcia Guirao and M. Lampart

R. Hric and M. Málek

J. Jian and J. Lü

M. Lampart

P. Oprocha 2x

L. Paganoni and J. Smítal

T. Sun

M. Štefánková

H. Wang and W. Song

L. D. Wang, Z. Y. Chu and G. F. Liao

**2007**

T. Arai and N. Chinen

K. Janková

G. F. Liao, L. D. Wang, X. D. Duan

M. Málek

P. Oprocha and P. Wilczynski 3x

P. Oprocha

R. Pikula

L. D. Wang, Z. Chu, and X. Duan

L. Wang, G. Huang and N. Wang

J. Xiong, J. Lü and F. Tan

**2008**

E. D’Aniello and T. H. Steele  
 J. S. Cánovas  
 X. F. Fu, Y. R. Liu and H. G. Gao  
 D. Kwietniak, P. Oprocha  
 G. Liao, Q. Fan and L. Wang  
 J. Lü, J. C. Xiong, F. Tang  
 P. Oprocha and M. Štefánková  
 L. Paganoni and J. Smítal  
 F. L. Pelayo  
 M. Štefánková  
 H. Wang, G. Liao and Q. Fan  
 L. Wang, S. Huan, G. Huang  
 L. Wang, X. Xing and Z. Chu  
 L. D. Wang, N. Wang, Z. Y. Chu

**2009**

F. Blanchard  
 X. Fu, Y. You  
 J. L. Garca Guirao, D. Kwietniak, M. Lampart, P. Oprocha and A. Peris  
 M. Lampart and P. Oprocha  
 C. M. Li and Q. J. Fan  
 G. Liao, Z. Chu and O. Fan  
 G. Liao, W. Wang and Q. Fan  
 H. Liu, L. Wang and Z. Chu  
 D. K. Ma and C. J. Hu  
 F. Martnez-Gimnez, P. Oprocha and A. Peris  
 P. Oprocha 3x  
 R. Garca Rubio  
 F. Tan and J. C. Xiong  
 H. Wang, G. Liao and Q. Fan 2x  
 H. Wang, Q. Fan and G. Liao  
 L. Wang, B. Li and Z. Chu  
 L. Wang, X. Xing and Z. Chen  
 X. Ma, B. Hou and G. Liao

**2010**

T. Bermúdez et all  
 X. C. Fu, Z. R. Liu and H. J. Gao  
 B. Hou, P. Cui and Y. Cao  
 V. Kornecká - Kurková  
 M. Lampart and P. Raith  
 R. Li 2x

L. Obadalová and J. Smítal  
P. Oprocha  
P. Oprocha and P. Wilczynski 2x  
M. Štefánková  
H. Wang, Q. Fan and G. Liao  
H. Y. Wang, J. C. Xiong and F. Tan  
L. Wang, S. Tang and Y. Chu