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# BAIRE ONE, GIBSON AND WEAKLY GIBSON REAL FUNCTIONS OF SEVERAL REAL VARIABLES

This presentation is based on results contained in a joint work [2] with Paul D. Humke.

In a recent paper [1] Humke and I explored the connectedness-preserving properties of various subclasses of the collection of real-valued, Baire one functions defined on $\mathbb{R}^n$. Also recently, Kenneth Kellum [3] investigated properties of Gibson and weakly Gibson real-valued functions defined on a connected space. Kellum selected these names in honor of his and our friend, the late Richard G. Gibson. In this note we consider the classes of Baire one, Gibson and weakly Gibson real-valued functions defined on $\mathbb{R}^n$. Specifically, we shall first observe that a Baire one function from $\mathbb{R}^n \to \mathbb{R}$ is a Gibson function if and only if it is continuous. Then we shall investigate the connectedness-preserving properties of Baire one, weakly Gibson functions from $\mathbb{R}^n \to \mathbb{R}$; that is, we shall explore which connected subsets of $\mathbb{R}^n$ such a function maps to intervals. We begin by specializing Kellum's definition to functions defined on $\mathbb{R}^n$.

**Definition 1.** A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be a *Gibson function* if for each open set $U \subseteq \mathbb{R}^n$, $f(\overline{U}) \subseteq \overline{f(U)}$. A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be *weakly Gibson* if for each connected open set $U \subseteq \mathbb{R}^n$, $f(\overline{U}) \subseteq \overline{f(U)}$.

Note that continuous functions are Gibson functions but that a Gibson function need not have any points of continuity as is readily evidenced by the characteristic function of the set of points having rational coordinates. However, if we consider Baire one, Gibson functions, then we have the following.

**Theorem 1.** *A Baire one function $f : \mathbb{R}^n \to \mathbb{R}$ is a Gibson function if and only if it is continuous.*

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Turning our attention to Baire one, weakly Gibson functions we first observe that when the domain is the real line, the class is identical to the class of Baire one, Darboux functions. As such, these functions map each connected set to a connected set. When we move to the higher dimensional case we can readily find examples of functions defined on $\mathbb{R}^n$ $(n > 1)$ which disconnect a line segment, for example. Finally, we can characterize the Baire one, weakly Gibson functions by specifying exactly which connected sets in $\mathbb{R}^n$ these functions map to intervals.

**Theorem 2.** *Let $n \geq 1$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be a Baire one function. Then $f$ is weakly Gibson if and only if for every open connected set $G$ and for every set $S$ for which $G \subseteq S \subseteq \overline{G}$, $f(S)$ is connected.*

## References

[1] M. J. Evans and P. D. Humke, *Generalizations of Young's Theorem to real functions of several variables*, Rend. Circ. Mat. Palermo, Ser. II, **58** (2009), 287–296.

[2] M. J. Evans and P. D. Humke, *Baire one, Gibson and weakly Gibson real functions of several real variables*, Rend. Circ. Mat. Palermo, Ser. II, **59** (2010), 47–51.

[3] K. R. Kellum, *Functions that separate $X \times \mathfrak{R}$*, Houston J. Math., to appear.