ON IDEALS WHICH COULD BE ASSOCIATED TO A POSET OF TREES

Let \((Q, \leq)\) be a poset. Consider the following types of ideals, which are associated to \((Q, \leq)\).

- \((q_0) = \{S \subseteq \bigcup Q : \forall p \in Q \exists q \leq p \ q \cap S = \emptyset\}\);
- \((q_1) = \{S \subseteq Q : \forall p \in Q \exists q \leq p \ \{y : y \leq q\} \cap S = \emptyset\}\);
- \((q_2) = \{S \subseteq Q^*: \forall p \in Q \exists q \leq p \ q^* \cap S = \emptyset\}\).

If \(Q\) is a collection of trees, e.g. an arboreal forcing condition like in [3], then meaning of \(\bigcup Q\) is cleared. Formally, trees are contained in \(Seq_X\) (finite sequences of elements from \(X\)) and any tree \(T \subseteq Seq_X\) one can identify with the set \([T] \subseteq X^{\omega}\) of all its infinite branches. \(Q^*\) denotes the collection of all maximal centered families which are contained in \(Q\) and \(p^* = \{U \in Q^* : p \in U\}\).

Results about:
- The ideal \((q_2)\) for \(([\omega]^{\omega}, \subseteq^*)\), see [1] and [2];
- \((q_0)\) for \(([\omega]^{\omega}, \subseteq^*)\) or the Mathias forcing conditions, compare [8] and [9];
- \((q_0)\) for the Silver forcing or \(n\)-Silvers forcing conditions, compare [6] and [7];

are generalized for system of of trees.

System of (Laver) trees where examined in [5] and [4]. Given

\[ < A_s \in [\omega]^\omega : s \in \omega^{<\omega} >= A \]

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define $p_s(\mathcal{A})$ to be the unique Laver tree such that the root of $p_s(\mathcal{A})$ is $s$ and for every node $t \supseteq s$ with $t \in p_s(\mathcal{A})$ we have that

$$\text{split}(p_s(\mathcal{A}), t) = A_t.$$ 

Define $\mathcal{A} \subseteq^* \mathcal{B}$ iff $A_s \setminus B_s$ is finite for all $s \in \omega^{<\omega}$, see [5], and define $\mathcal{A} \prec^* \mathcal{B}$ iff $A_s \subseteq B_s$ for all but finite many $s \in \omega^{<\omega}$, see [4]. The poset $\subseteq^*$ is separative, so one can directly adopt Base Matrix Tree Theorem and Kulpa - Szymański Theorem similarly as in [1], [2]. The poset $\prec^*$ is not separative, so it need a non separative version of Base Matrix Tree Theorem. Counter-examples of systems of trees without properties needed for non separative version of the Base Trees Theorem are given, too.

References


