Differentiability and the lip f function

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Given a continuous function $f : \mathbb{R} \to \mathbb{R}$, the upper and lower scaled oscillation functions Lip f and lip f are defined as follows:

$$\text{Lip } f(x) = \limsup_{r \to 0^+} \frac{L_f(x, r)}{r},$$

$$\text{lip } f(x) = \liminf_{r \to 0^+} \frac{L_f(x, r)}{r},$$

where

$$L_f(x, r) = \sup\{|f(x) - f(y)| \colon |x - y| \leq r\}.$$

We also define $N_f = \{x \in \mathbb{R} \mid f \text{ is not differentiable at } x\}$ and we let Lip $\mathbb{R}$ (lip $\mathbb{R}$) be the set of all functions $f : \mathbb{R} \to \mathbb{R}$ such that Lip $f(x) < \infty$ (resp. lip $f(x) < \infty$) for all $x \in \mathbb{R}$. Then the following result follows from a theorem of Zahorski and the Rademacher-Stepanov Theorem.

Theorem 0.1 *$E = N_f$ for some $f \in \text{Lip } \mathbb{R}$ if and only if E is a $G_{\delta\sigma}$ set and $|E| = 0$.*

In my talk I will look at analogues of this theorem with the condition $f \in \text{Lip } \mathbb{R}$ replaced by $f \in \text{lip } \mathbb{R}$.