Exploring Credit Card Debt

Problem: You have a \$1000 balance on a credit card that charges 1.5% interest per month. Suppose you can afford to pay \$30 per month, and that you do not make any other charges on this credit card.

TI-89 and TI-92 Calculator Suggestions:

You will be asked to to calculate a table of values and to construct a graph for a sequence. To do this, you need to change the graphing mode and sequence settings as follows:

- Mode: In the *Mode* dialog box, set the graph mode to *sequence.* (See pages 33-34 in your TI-89 manual.)
- Table Setup: In the *Table Setup (TblSet)* dialog box, be sure the *Independent* setting is on *Auto.* Then enter 0 as the *tblStart* value, 1 as the $\Delta\, tbl$ value. Be sure the *Graph* $\leftrightarrow$ *Table* setting is on *Off.*

Exploring the Problem :

This problem can be modeled using a *difference equation*, i.e., an equation of the form below where $u(n)$ represents the amount owed at the end of n months.

$$\Delta u(n) = u(n) - u(n-1)$$

1. Assume your beginning credit card balance is \$1000.

   (a) Write an equation for $\Delta u(n)$ that shows how the amount you owe changes from month $n-1$ to month n. Note: Your equation will necessarily include $u(n-1)$ in a way somewhat analogous to the example below.

An Example Problem:	**Credit Card Problem:**
$\Delta u(n) = u(n-1) + 10$	

   (b) Using the example solution below as a guide, solve your difference equation to find a function that gives the value of $u(n)$ as a function of $u(n-1)$. (Such a function is called a *recursive* function.)

	Example Problem:		**Credit Card Problem:**
If	$\Delta u(n)$	$= u(n-1) + 10,$	
then	$u(n) - u(n-1)$	$= u(n-1) + 10.$	
So	$u(n)$	$= 2u(n-1) + 10.$	

   (c) Enter the function you found in part (b) as the sequence $u1$ in the **Y=** Editor of your calculator and enter the value 1000 as your initial value $ui1$.

   For the example above, you'd enter $u1 = 2 \cdot u1(n-1) + 10$ and $ui1 = 1000$

   (d) Investigate how the value $u(n)$ changes both by graphing the sequence and looking at a table of values for the sequence.

   Graph: To graph the sequence, begin with the *Window* settings shown below:
   nmin =0 nmax = 10 plotStrt =1 plotStep=1 xmin=0 xmax = 10 x scl =1, etc.

   Table: Observe the calculated values for $u(n)$ for consecutive values of n. To see additional values, use the arrow key to scroll down through the table.

   (e) When do you pay off the balance?

2. Assume your beginning credit card balance is \$2500. Use a similar investigation to determine when you pay off the balance in this case.

3. For what starting balance will you exactly pay the interest each month? This value is called the *equilibrium value*. Note: This is an unstable equilibrium because any other starting value causes the balance to diverge from the equilibrium.

4. Using substitution in the formula you found for $u(n)$, verify by hand that the equation $u(n) = (1.015)^n C + 2000$ gives a solution to the credit card equation that you found in activity 1(b). Note: For one side of the equation you'll need to replace n by $n-1$ to the difference equation that you wrote in problem 1. What is the value of C when the initial balance is $1000?

5. Using this value of C, verify numerically that $u(n) = (1.015)^n C + 2000$ is a solution to the difference equation that you wrote in problem 1. (Hint: As before, enter your recursive function $u1 = u(n)$, with initial value $ui1 = 1000$. Also enter $u2 = (1.015)^n C + 2000$ with C replaced by the value you found in problem 4 (For $u2$, you do not need to specify an initial value.) Then observe a table giving values of both $u1$ and $u2$ for consecutive values of n.

6. BONUS: How could you find the solution given in problem 4?

7. DISCUSSION: What does all this have to do with calculus?

Background Information on Credit Card Debt

The following information is quoted from the article *Undergraduate Credit Card Debt* in the June 15, 2001 **Chronicle of Higher Education.**, pp. A35-A36.

Undergraduate Credit Card Debt		
	1998	*2000*
% who have credit cards	67%	78%
% who have 4 or more credit cards	27%	32%
% with balances between $3,000 and $7,000	14%	13%
% with balances exceeding $7,000	10%	9%
Average credit card debt	$1,879	$2,748
Median credit card debt	$1,222	$1,236
Average number of credit cards	3.5	3.0
Percent paying off balance in full each month		56%

"A student who makes only the minimum monthly payment (usually 2 or 3% of the balance) on a card with an 18% annual percentage rate and a balance of $2,478 will end up paying as much in interest as she originally charged. It would take her about 15 years to pay off the balance.

"According to a recent study by Elizabeth Warren, a professor at Harvard Law School, an estimated 120,000 people under the age of 25 filed for personal bankruptcy in 2000.

"The issue of student debt is complicated by the industry's major support of politicians and educators. Industry groups gave more than $9 million in campaign contributions in 2000, according to the Federal Election Commission. That was just slightly more than the tobacco industry gave and double the contributions made by gun rights advocates last year. President Bush's biggest campaign donor last year was MBNA America Bank, one of the two largest distributors of credit cards on college campuses, and a proponent of the bankruptcy legislation.

"The most widespread example of the business partnerships between academe and the credit card industry are so-called monopoly contracts. In its current contract with the University of Oklahoma, for example, First USA will pay the University $13 million over 10 years for the exclusive right to market Mastercards and Visa cards to students, alumni, and employees, and to issue credit cards bearing the University's name. The bank will also give the school .4% of every purchase charged with the card."

Revision by JNC for St. Olaf students from a Carroll College Activity, August 24, 2001