

Answers to Test 1 Review Problems

1. Let $F(x) = \int_0^x f(t) dt$ where $f(t)$ is the function graphed on your review sheet.

(a) Give exact values for the following:

i. $F(2) = -2$

ii. $F(-5) = .5$

iii. $F'(-4) = -1$

(b) On which subinterval(s) of $[-5, 5]$, if any, is F increasing? $(-3.5, 0)$ and $(3.5, 5)$

(c) On which subinterval(s) of $[-5, 5]$, if any, is the graph of F concave down? $(-3, 2)$

3. Evaluate each of the integrals below using either substitution or parts as necessary. Be sure to indicate your u and du values for substitution and your u , dv and dv values for parts. You may check your answers with your calculator.

(a) $\int \frac{(\ln x)^2}{x} dx = \frac{(\ln x)^3}{3} + C$

(b) $\int_1^4 x(x-3)^2 dx = \int_1^4 x^3 - 6x^2 + 9x dx = \frac{21}{4}$

(c) $\int_1^8 \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx = \frac{72\sqrt{2} + 97}{2}$

(d) $\int e^x \sqrt{e^x + 5} dx = \frac{2(e^x + 5)^{1.5}}{3} + C$

(e) $\int_0^2 \frac{x+3}{x+10} dx = \int_0^2 \left(\frac{x}{x+10} + \frac{3}{x+10} \right) dx = .7237$

(f) $\int \frac{\arctan(x)}{1+x^2} dx = \frac{(\arctan x)^2}{2} + C$

(g) $\int \frac{\cos x}{\sqrt{1-\sin^2 x}} dx = \pm x + C$

4. Let $I = \int_1^2 f(x) dx$ where $f(x) = \sin(\ln x)$

(a) Sketch a graph showing $f(x)$ and illustrate the *left sum* approximation L_6 for I . Then use “ Σ ” notation to write out L_6 .

$$L_6 = \sum_{i=0}^5 \sin(\ln(1 + (i/6)))(1/6)$$

(b) Rank the values of L_{100} , R_{100} , and T_{100} in order of increasing size and explain, using sketches, how your ranking can be determined using only graphical information. Which of these approximations will be most accurate? Why?

$$L_{100} < T_{100} < I < M_{100} < R_{100} \quad M_{100} \text{ will be most accurate.}$$

(c) $L_{10} = .33726$; $|I - L_{10}| < .05$; (Using $K_1 = 1$)

(d) For R_n , $n \geq 100$; for M_n ; $n \geq 3$ (Using $K_2 = 1$)