Math 126C May 18, 2005

Test 1 Procedure:

The test will be taken under the honor system. You may bring and use **only** the following items: a pencil (or pen), and a graphing calculator.

Test Coverage: Sections 3.4, 5.1-5.7, 6.1-6.2

1. **Inverse Trig Functions:**
   (a) Domain and range of inverse trig functions, particularly, $arcsin(x)$ and $arctan(x)$
   (b) Simplifying expressions like $cos(arcsin(x))$
   (c) Derivatives of $arcsin(x)$ and $arctan(x)$
2. Understand and explain (using diagrams) the following concepts:
   (a) Interpretation of a definite integral as signed area
   (b) Definition of integral as a limit of Riemann Sums (p. 353)
   (c) Fundamental Theorem of Calculus (FTC), both versions (Theorems 3 & 4 - p. 322, 325)
   (d) Understand the geometric reasoning used to verify Version 1 of the FTC
   (e) Properties of definite integrals (see p. 306-307, 311)
   (f) Average value (p. 309)
3. Evaluation of integrals:
   (a) Be able to evaluate indefinite integrals using recognition of the antiderivative, substitution, and your calculator.
   (b) Be able to evaluate definite integrals exactly using the previous techniques and the FTC.
   (c) Be able to evaluate and/or approximate definite integrals when the integrands, $f(x)$, are given graphically
   (d) Be able to give approximations of definite integrals when the values of the integrands are given numerically (e.g., in tables)
   (e) Be able to set up and find appropriate definite integrals to measure total (net) change
4. Approximation of definite integrals using L_n, R_n, M_n, T_n (Sec. 6.1, 6.2)
   (a) Be able to geometrically illustrate and give written interpretations for each of these.
   (b) Be able to use Σ notation to represent these approximation sums using n equal subintervals.
   (c) Be able to evaluate L_n, R_n, T_n and M_n using your calculator.
   (d) Be able to use error bound theorems for $|I - L_n|,\ |I - R_n|,\ |I - T_n|,\ |I - M_n|$ both to determine the possible error in approximating with a known n and to determine how large n must be to guarantee any given accuracy in an approximation. Note: The formulas in these theorems will be supplied with the test.

Review Problems

1. Let $F(x) = \int_0^x f(t)\,dt$ where $f(t)$ is the function graphed.
   (a) Give exact values for the following:
      i. F(2)
      ii. F(-5)
      iii. F'(-4)
   (b) On which subinterval(s) of $[-5, 5]$, if any, is F increasing? Explain.
   (c) On which subinterval(s) of $[-5, 5]$, if any, is the graph of F concave down? Explain.

2. Give a precise definition of the definite integral $\displaystyle\int_a^b f(x)\,dx$. Be sure to use a Riemann sum in your definition and explain all the symbols involved.

3. Evaluate each of the integrals below using either substitution or parts as necessary. Be sure to indicate your u and du values for substitution and your u du, v and dv values for parts. You may check your answers with your calculator.

   (a) $\displaystyle\int \frac{(ln\,x)^2}{x}\,dx$ (b) $\displaystyle\int_1^4 x(x-3)^2\,dx$ (c) $\displaystyle\int_1^8 \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx$

   (d) $\displaystyle\int e^x\sqrt{e^x+5}\,dx$ (e) $\displaystyle\int_0^2 \frac{x+3}{x+10}\,dx$ (f) $\displaystyle\int \frac{arctan(x)}{1+x^2}\,dx$

   (g) $\displaystyle\int \frac{cos\,x}{\sqrt{1-sin^2x}}\,dx$

4. Let $I = \displaystyle\int_1^2 f(x)\,dx$ where $f(x) = sin(ln\,x)$
   (a) Sketch a graph showing $f(x)$ and illustrate the *left sum* approximation L_6 for I. Then use "Σ" notation to write out L_6.
   (b) Rank the values of L_{100}, R_{100}, and T_{100} in order of increasing size and explain, using sketches, how your ranking can be determined using only graphical information. Which of these approximations will be most accurate? Why?
   (c) What can you say about the error committed in estimating I by L_{10}? Be sure to use one of the error bound theorems to find your answer.
   (d) Which value of n will guarantee that R_n approximates I within $\pm.005$? Which value of n will guarantee that M_n approximates I within $\pm.005$?

Other Review Problems

P. 365: 1-4, 11, 26-44, 50, 52, 53, 67-69