Math 126C May 18, 2005

Test 2 Procedure:

The test will be taken under the honor system. You may use **only** a pencil (or pen), and a calculator.

Test Coverage: Sections 4.2, 7.1-7.2, 7.4-7.5, 8.1, 9.1-9.2

1. Evaluation of limits of indeterminant forms using L'Hopital's Rule (Sec. 4.2)
   Be able to identify determine the limit for each of the following forms using L'Hopital's Rule: $\frac{0}{0},\ \frac{\infty}{\infty},\ \infty-\infty,\ 0\cdot\infty,\ 0^0,\ \infty^0\ 1^\infty$
2. Integration by Parts (8.1)
   Be able to evaluate indefinite integrals using recognition of the antiderivative, substitution, parts, and your calculator.
3. Applications of Integration (Sec. 7.1-7.2, 7.4-7.5)
   For each application below, be able to (1) explain how the application leads to an integral and (2) set up and evaluate the appropriate integral.
   (a) Area between curves
   (b) Volume of solids - including solids of revolution and more general solids where the area of the cross-sections is given by a function $A(x)$.
   (c) Arc Length
   (d) Solving Separable DE's modeling exponential growth, logistic growth and warming (A chart of general forms and their solutions will be provided.)
   (e) Finding present value.
4. Taylor Polynomials (Sec. 9.1-9.2)
   (a) Know the definition of Taylor polynomials and be able to find a Taylor (or Maclaurin) polynomial for a function $f(x)$
   (b) Know how to use Taylor's theorem to find an upper bound on the error in approximation of $f(x)$ by $P_n(x)$ where the later is a Taylor polynomial. Note: The formula in the theorem will be provided.

Review Problems

1. Without using your calculator, evaluate
   (a) $\lim_{x\to\infty} x^2e^{-x}$ (b) $\lim_{x\to 0}\frac{sin(2x)}{x}$ (c) $\lim_{x\to\infty}(1+\frac{1}{x})^x$
2. Find the volume of the solid whose base is the region enclosed by $y=1-x,\ x=0$, and $y=0$ and whose cross-sections perpendicular to the x-axis are semicircles with diameter on the base. Include in your solution (1) a sketch of the base of the solid showing where you cut out a think cross-sectional slice; (2) a sketch of your cross-sectional shape; (3) formulas for the cross-sectional area and volume of the slice; (4) an explanation of why an integral is used to find the actual volume; and (5) evaluation of the integral.
3. Let $f(x)=x^2$ for $0\le x\le 1$
   (a) Use a graph to explain why the arc length of $f(x)$ over this interval is great than $\sqrt{2}$.
   (b) Find the actual arc length of $f(x)$ over this interval.

4. Let $\sum_{k=1}^{10} \frac{(x-2)^k}{7k}$ be a Taylor polynomial for a function $f(x)$. What is the value of the eighth derivative $f^{(8)}(2)$?

5. Find a solution to the initial value problem $y' = \frac{x}{\sqrt{y}}, \; y(0) = 1$.

6. Let $f(x) = sin\, x$

   (a) Use your calculator to find the Taylor polynomial $P_3(x)$ for $f(x)$ based at $x = \pi$.

   (b) Use Taylor's Theorem to find an upper bound for the error committed by approximating $sin(\frac{9\pi}{8})$ by $P_3(\frac{9\pi}{8})$.

7. What amount of money would your family have needed to invest at 6% interest at the time of your birth to pay the 2004-2005 comprehensive fee of $30,950.

8. Let R be the region in the first quadrant enclosed by $y = x^2$ and $y = 4$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated around the line $x = 2$.

9. Evaluate each of the integrals below using either substitution or parts as necessary. Be sure to indicate your u and du values for substitution and your $u\; du$, v and dv values for parts. You may check your answers with your calculator.

   (a) $\int \frac{(ln\, x)^2}{x}\, dx$ (b) $\int arctan\, x\, dx$ (c) $\int x\, ln\, x\, dx$

   (d) $\int_0^2 x\, cos(x)\, dx$ (e) $\int_0^2 x\, cos(x^2)\, dx$

10. Let R be the region enclosed by $y = 4x$, $x = 0$, and $y = 8$ and let S be a solid with base on the region R. The cross sections of S perpendicular to the base are squares with one side on the base.

    (a) Draw a sketch of the region R showing where you cut out a thin cross-sectional slice to obtain a cross-section of the solid S. Then draw a sketch showing a cross- sectional shape of S.

    (b) Find the volume of the solid S by setting up formulas for the cross sectional area (ΔA) and the cross sectional volume (ΔV) and using integration.

11. Assume that the temperature $y(t)$ of a cup of coffee in degrees Farenheit at time t minutes is given by the differential equation: $y' = -.0.1\,(y - T_0)$ where T_0 represents the initial temperature of the coffee. If you place a cup of coffee at 120^o in a refrigerator with inside temperature of 35^0, how long will it be until your coffee has cooled to 90^0?