

Test 3 Procedure:

You may use **only** a pencil/pen, a calculator and one 5×8 card of notes.

Test Coverage: Sections 10.1-10.2, 11.1-11.5**Improper Integrals (10.1-10.2)**

1. Be able to identify and evaluate both types, using the limit definitions.
2. Be able determine if an improper integral is convergent or divergent.
3. Be able to use the Comparison Theorem (p. 531).

Infinite Sequences (11.1)

1. Notations and definitions: infinite sequence, general term, limit, convergence, divergence, upper and lower bounds.
2. Theorems: L'Hopitals' Rule, Squeeze Principle, Convergence of bounded monotone sequences

Infinite Series (11.2-11.6)

1. Terminology and notation: infinite series, general term, partial sum, nth remainder, limit or sum.
2. Definitions: convergence, absolute convergence, conditional convergence, divergence.
3. Series with Constant Terms: Be able to apply and explain the use of convergence tests for ...
 - (a) All series using
 - recognition as a p-series (know which ones converge and diverge), or as a geometric series (know which ones converge and diverge, and the sum formula)
 - general (nth) term test for divergence (Theorem 6, p. 563)
 - (b) Series with non-negative terms
 - Comparison test
 - Ratio test
 - Integral test (Be able to use to find upper and lower bounds for S , bound for $R_n = S - S_n$.)
 - (c) Series with some negative terms
 - Absolute convergence test
 - Alternating series test (Be able to use to find upper and lower bounds for S , bound for $|R_n| = |S - S_n|$)
4. Power Series:
 - (a) Terminology and notation: power series, base point, interval and radius of convergence.
 - (b) Be able to determine the open interval of convergence and radius of convergence.
 - (c) Be able to determine convergence or divergence at endpoints of interval of convergence.

Review Problems

1. Determine whether each of the following converge or diverge. For those that converge, find upper and lower bounds for the limit of the series. In each case, show your work and name the convergence test you are using.

(a) $\sum_{k=0}^{\infty} \frac{1}{2^k + 7\sqrt{k}}$ (b) $\sum_{j=0}^{\infty} \frac{j!}{100^j}$ (c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[3]{k}}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

2. A series $\sum_{k=1}^{\infty} a_k$ has partial sums given by $S_n = \frac{3n+2}{n+1}$. Justify your answers for the following:

(a) Does the series converge? (b) Find values of a_1 and a_5 (c) Find the value of R_5 . (d) Find $\lim_{n \rightarrow \infty} a_n$

3. Suppose that the radius of convergence of the series $\sum_{k=0}^{\infty} a_k(x+3)^k$ is 10. Does the series converge when $x = 8$? when $x = -13$? Explain.

4. Consider the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k 2^k}$

- (a) Find the radius and open interval of convergence for the series.
(b) Does the series converge at the endpoints of the interval? Show your work in answering this question.

5. Consider the series $\sum_{k=1}^{\infty} \frac{1}{k^4 + 2k}$

- (a) Prove that this series converges. Be sure to explain your reasoning, naming any convergence tests used.
(b) Find a value of n such that the partial sum S_n for this series approximates the actual sum within .01. Explain your work.

6. Suppose that the series $\sum_{j=0}^{\infty} b_j$ converges where $b_j \geq 0$ for all j . Does the series $\sum_{j=0}^{\infty} (-1)^j b_j$ converge? Explain your answer.

7. Let $\{a_k\}_{k=0}^{\infty}$ be an increasing sequence such that $a_1 > 0$ and $a_k \leq 100$ for all k .

(a) Does $\{a_k\}_{k=0}^{\infty}$ converge? (b) Does $\sum_{k=0}^{\infty} a_k$ converge? Explain your answers.

8. Explain why the integrals below are improper and determine whether the integrals converge or diverge.

(a) $\int_1^{\infty} \frac{1}{x^3 + 4\sqrt{x}} dx$ (b) $\int_{-1}^3 \frac{1}{x-2} dx$

9. Assume that $P(x) = \sum_{k=0}^{\infty} \frac{(-.5)^k (x-7)^k}{k!}$ is a Taylor series for the function $h(x)$.

(a) Find the radius and open interval of convergence for $P(x)$. (b) Find the sixth derivative $h^{(6)}(7)$