Answers to Final Review Problems

1. (b) 7 (c) 6/7 (d) (18/5, 24/5, 0)

2. (a) 6, 4
   (b) Nul A: $\{[-2,1,0,0,0,0],[1,0,-2,-1,1,0],[0,0,-1,-2,0,1]\}$;
   Col A: $\{[1,1,1,3],[0,1,0,1],[1,1,2,4]\}$
   (c) No; No (d) $\{[-2x_2+x_5, x_2, -2x_5+1, -x_5+2, x_5]\}$

3. $det(A) = \lambda_1\lambda_2\cdots\lambda_n$

4. It is not a subspace, since it is not closed under either scalar multiplication or addition.

5. $A^T A = I$.

6. (a) $c_{31} = 11$, $c_{32} = 4$, $c_{33} = 6$ (b) 1 (c) $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

7. (a) 5, 7 (b) 5 (c) yes; maybe (d) 3 (e) 7, 5

8. (a) W is a subspace of $\Re^3$ since it is the span of vectors in $\Re^3$.
   (b) $\mathbf{v} = 2(2,0,1) + 3(-1,0,1)$ (d) $[2,3]_B$
   (e) (ii) $[T]_B = \begin{bmatrix} 7/3 & -2/3 \\ 5/3 & -4/3 \end{bmatrix}$ (iii) $[8/3, -2/3]_B$

9. (a) $[1,-7]$ (b) $B = \{(1,1),(1,-1)\}$ (c) $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix}_B = \begin{bmatrix} -3 \\ 4 \end{bmatrix}_B$

10. (a) $A = \begin{bmatrix} 5 & -2 \\ -6 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1/2 & 0 \\ -1/6 & 1/3 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -6 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

11. (b) One possible basis is $\mathcal{B} = \{1+2t^2, t\}$
    (c.i) Answer will depend on basis $\mathcal{B}$ used; for basis given above, the matrix is $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$
    (c.ii) $(-1, 8)$.