Experiment O5

THE SPEED OF LIGHT

References:
Jenkins and White, *Optics*.

Object:
This experiment uses the method of Foucault (1850) to measure the speed of light by determining the time it takes a light beam to travel from one end of the laboratory to the other and back. The round trip time is obtained using the fact that the mirror turns through a small but measurable angle during that interval.

The first determinations of the speed of light came from astronomical observations. (How?) The method begun by Foucault was repeated with many variations, evacuated chambers, longer paths, multiple mirrors, etc., all done to improve the precision. Now other methods give better precision. (See reference above.)

Background:
The optical arrangement is shown in Fig. 1. The beam from a laser passes through a slit, from there to the rotating mirror, then the lens and finally to the distant fixed mirror. Position the slit so that the image of the slit, due to the lens, is focused on the fixed mirror. When this is accomplished, the beam can be reflected by the fixed mirror and will retrace itself, coming to a focus for a second time in the plane of the slit. We couldn't see the final image without sticking your head into the beam. Thus a beam splitter is inserted into the beam as shown in Fig. 2.

If the mirror has rotated during the passage of the beam the image of the slit will be displaced by a small amount \( s \). The reason for this is that in the time that the light has traveled the distance down to the stationary mirror and back to the rotating mirror the rotating mirror will have turned through an angle \( \theta \). The light beam will be deflected through an angle \( 2\theta \). The factor of two comes from an application of the law of reflection, a point that should become clear if you redraw an enlarged view of Fig. 2. The displacement of the
spot $s$ is then given by:

$$s = (2 \Delta \theta) r$$

where $r$ is the distance from the rotating mirror to the scale where $s$ is placed. The angle through which the mirror turns is given by

$$\Delta \theta = \left( \frac{d\theta}{dt} \right) \Delta t = \omega \Delta t$$

where $\omega$ is the rotational velocity in radians per second. If we call the distance from the rotating mirror to the stationary mirror $b$, then

$$\Delta t = \frac{2 b}{c}$$

From this it follows that

$$s = 2 \omega \left( \frac{2 b}{c} \right) r$$

Since $s$ should be proportional to $\omega$, the ratio of $s/\omega$ should be a constant. This assumes that $s=0$ when $\omega=0$, a point which is difficult to obtain. A better way to get this ratio is to measure $s$ for several values of $\omega$ over as wide a range of $\omega$ as possible. A linear fit of a plot of $s$ as a function of $\omega$ gives $\Delta s/\Delta \omega$ which is then equal to $s/\omega$. From the result of this analysis we can write an expression for $c$ in terms of measurable quantities. We get

$$c = \frac{4 b r}{(\Delta s / \Delta \omega)}$$

The displacement $s$ that is obtained for the top speed of the mirror is a few millimeters. To detect such a deflection with any precision, the size of the image at the scale has to be very small. There are two conditions have to be met to make it as small as possible. The first comes from the fact that as the rotating mirror turns the spot may may become smeared out. As will be discussed below, this can be accomplished by putting the rotating mirror in the focal plane of the lens.

The second effect comes from the fact that the slit cannot be perfectly imaged by the lens. The minimum size of the slit's image on the fixed mirror and its reflection back in the plane of the slit is ultimately diffraction limited. The rotating mirror acts as a 'slit' producing a Fraunhofer diffraction pattern instead of a sharp image of the slit. Since this diffraction pattern narrows as the 'slit' widens, we want the beam as wide as possible. The best we can do is have it fill the width of the rotating mirror. To accomplish this, you should narrow the slit width until the central maximum of its diffraction pattern is as wide as the rotating mirror when it reaches the mirror. Beyond that, narrowing the slit only reduces the intensity of the beam.

**Procedure:**
According to the discussion above, we have to meet two optical conditions: 1) The lens must form an image of the source slit on the fixed mirror. Then, if the fixed mirror is adjusted properly, the returning beam will pass back through the lens and be imaged on the scale. The image should be the size of the original slit though diffraction limited. It is hoped that the spot will be small enough that it should be possible to see the image shift when the rotating mirror is turning. 2) The focal point of the main lens must be in the focal plane of the rotating mirror. This will guarantee that, as the beam sweeps across the lens, the light will reflect off of the stationary mirror and retracing their path on the return except for the shift caused by the turning of the rotating mirror during the round trip. This is shown in Fig. 3. Each of the three paths shown in the figure will be shifted by the same amount and thus will not appear to be smeared out. These conditions make the alignment and positioning of the elements of the optical system rather critical, and a careful procedure is necessary to avoid having to repeat adjustments later. The following sequence is suggested:

1. Set up the laser, the slit (wide open), the rotating mirror (while turned off), and the fixed mirror so that the beam hits the center of the rotating mirror, passes through the center of the lens and hits near the center of the fixed mirror. Adjust the distant mirror to reflect the beam back through the lens, but slightly off to the side of the beam coming from the laser.

2. Orient the main lens so that the beam passes through its optical center with the beam lined up with the optical axis. You can accomplish this by taking advantage of the fact that you get two partial reflections from the lens surfaces. When the alignment is correct, the two reflected spots will lie on the rotating mirror. One of the spots is larger than the other because one surface of the lens is concave and the other is convex.

3. Use auto-collimation to locate the lens so that the rotating mirror is in the focal plane of the lens. To do this, take the small, short focal length lens and temporarily place it between the laser and the rotating mirror. Temporarily remove the slit. Now move the short focal length lens so that it focuses the laser beam to a small spot on the surface of the rotating mirror. The beam will spread out and pretty much fill the main mirror. When the rotating mirror is in the focal plane of the main lens, you will have a parallel beam between the main lens and the fixed mirror. This will be reflected back through the lens and come to a sharp focus near the rotating mirror. A dusted piece of glass is very convenient to use at this point as you can see both the beam emerging from the rotating mirror and the returning beam converging on the mirror. You can translate the main mirror horizontally until the two spots are in the same plane, i.e., the plane of the rotating mirror. The large lens is in its final position and the small lens can be removed.

4. Replace the slit and position it so that its image lies on the distant fixed mirror. Here
again the dusted piece of glass is useful. You can see the beam converging to a focus on the fixed mirror and the reflection diverging from the focus. If the slit is not positioned correctly, the image of the slit will be in front or 'behind' the fixed mirror and this becomes quite apparent when you compare the two spots on the glass as you move it closer or further from the fixed mirror. Now orient the fixed mirror so that the beam retraces itself as nearly as possible.

5. If the rotating mirror and the fixed mirror are properly oriented, the beam should now be retracing itself and coming to a focus a second time near the slit. The partially reflecting glass beam splitter will produce two beams, one from each side of the glass. You can use either of these beams. Position the measuring scale where the image of the slit is the sharpest even if the position isn't at the same distance from the rotating mirror as the slit itself. (Use a neutral density filter to cut the intensity of the beam.) The eyepiece is simply a magnifier to give you a closer view of the scale. Adjust it so the scale appears in sharp focus.

6. A light detector can be used to register an electrical pulse each time the rotating mirror sweeps the beam across it. Get the detector in as close to the rotating mirror as convenient, while not blocking off the beam. The time interval between pulses can be measured with a scope or even better with an electronic counter to get a value for $\omega$. Remember that there are two sides to the mirror so that there will be two pulses per revolution.

The experiment can be completed in good order by starting up the rotating mirror and measuring the shift of the spot as you vary the frequency. You will perhaps be pleasantly surprised when you try to fit this to a straight line to find out how precise this measurement turns out to be. Use the results to determine the speed of light.