F09 Math 242 HW Solution

HW # 2

1. a) System 1 is irreducible and aperiodic. States 4 + 5 are absorbing. This is ascertained by raising Λ_1 to a high power. Each column approaches $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2/3 \\ 1/3 \end{bmatrix}$

States 3 and 4 do not communicate with states 1, 2 or 3.

3

b) System 2 is irreducible and aperiodic. We see this because Λ^4 has no zero entries.

2) Transpose this

curr \ next	A	B	C	D	E	F	G	H	I	J	K
A	1/6	1/6	1/3	1/6	1/12		1/12				
B	1/6		1/6	1/6	1/6	1/6	1/6				
C	1/12			1/12	1/6	1/3	1/3				
D					1/12	1/3	5/12	1/6			
E	1/12					1/6	1/3	1/3	1/12		
F	1/6						1/6	1/3	1/6	1/6	
G	1/6							1/6	1/6	1/3	1/6
H	1/4								1/2	1/3	1/3
I	1/3	1/6								1/6	1/3
J	1/3	1/3	1/6								1/6
K	1/4	1/3	1/3	1/12							

10 pts

transpose

2) continued.

First, we see if this matrix is aperiodic and irreducible. Using R we see A^4 has no zero entries.

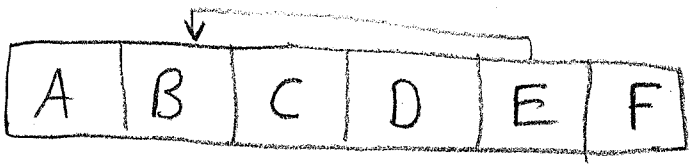
To find the long term steady state probabilities, we can find the eigenvector of A corresponding to $\lambda = 1$ with positive entries that sum to 1.

$\pi_A = 0.178$
$\pi_B = 0.093$
$\pi_C = 0.115$
$\pi_D = 0.061$
$\pi_E = 0.055$
$\pi_F = 0.083$
$\pi_G = 0.126$
$\pi_H = 0.077$
$\pi_I = 0.046$
$\pi_J = 0.089$
$\pi_K = 0.077$

In decreasing order

A
G
C
B
J
F
H
K
D
E
I

3) 10pt



Version 1

From / To	A	B	C	D	F	Lost
A		1/3		1/3	1/3	
B	1/3	1/3	1/3			1
C	1/3	1/3			1/3	
D	1/3	1/3	1/3			
F			1/3	1/3		
Lost		1/3	1/3	1/3	1/3	

From / TO	A	B	C	D	F	
A				1/3	1/3	
B	1/3	1/3	1/3	1/3	1/3	
C	1/3	1/3			1/3	
D	1/3	1/3	1/3			
F			1/3	1/3		

Note: A stat for E.

3) cont'd

Version 1 Steady State

A	0.088
B	0.250 + 0.250 = 0.500
C	0.147
D	0.162
F	0.103

Version 2

A	0.118
B	0.333
C	0.196
D	0.216
F	0.137

4) Spl. Yes, such papers exist in the literature.