Who Owns School Mathematics?

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Now that the Standards are visible, people are noticing them. They are no longer just a banner behind which the converted can march, but now they are also a target at which critics can aim their arrows. One might wonder what's worse: issuing a document that everyone ignores, or publishing a report that everyone fights about.

I ask in my title "Who owns school mathematics?" Perhaps one might ask, instead,

- Who should own school mathematics?
- Who thinks they own school mathematics?
- Who wants to own school mathematics?

Outsiders often get the impression that NCTM believes that they own school mathematics. After all, it is their Standards, promoted by their journals, their conferences, their workshops, and their publications. Politicians and education pundits canonize NCTM's Standards. If the Standards are the bible of the current reform movement, then NCTM seems cast in the role of Pope.

This analogy may be hyperbole--and risky too--but the verbal battles that have erupted in recent years about mathematics education bear uncanny resemblance to disputes in the history of religion. In today's landscape of mathematics education, one can easily find many different religions--even different gods. For some, the ultimate goal is enlightenment, for others, employment.

One can find mathematical monotheists who insist on one goal for all, and polytheists who support different goals for different folks. Puritans preach strict adherence to traditional preparation for scientific careers; Unitarians preach inclusiveness, seeking to make mathematics ennobling for all; Catholics seek to follow their Pope, while Muslims and Jews follow different traditions based on their versions of holy writ. Fundamentalists, too, can be found in every mathematical religion--in those who know what is right and want to preserve their vision for future generations.

We are familiar with the consequences of centuries of religious warfare in Northern Ireland and India, in the Balkans and the Middle East. Although the culture of mathematics education is measured in decades not centuries, we can see some of the same characteristics: deep-seated and seemingly ineradicable disagreements on the means and ends of school mathematics.

We also know that the only route to accord is serious dialogue based on mutual respect and understanding of others' views.

Unfortunately, mathematicians' instincts when challenged are to argue: to prove that their assertions are correct and that conflicting conjectures must be wrong. This is also the
theologians' instinct--whether it be a rabbi or a bishop, an ayatollah or a pope. Argument is a powerful tool of mathematics because in an axiomatic system it can indeed pave the way to truth. But to achieve consensus in mathematics education, argument is less important than understanding and mutual respect.

Everyone owns school mathematics, and everyone's views deserve to be heard and understood.

Voices of Concern

Critics of the Standards have found their voices--voices that, unfortunately, often seem shrill and rigid. You've heard them all, I'm sure. It comes with living in California that you are on the front wave of everything--including both development of Standards and public ridicule of them. Although some of the critics are strident, their messages are nonetheless worth hearing. Even more important, we need to listen to those who are perhaps more thoughtful yet less often heard, being drowned out by the cacophony of high-profile politically-inspired arguments.

In the interests of helping move the discussion from argument to understanding, I will in what follows seek to express the major concerns of many of the constituents of school mathematics--erstwhile owners, if you will, who seek their just claim to part of the territory.

First let me characterize (and perhaps caricature) some of these divergent concerns:

- Those who are worried about the shift from a goal of mathematics for scientists to mathematics for all.
- Those who recoil at the "psychobabble" of educational jargon (e.g., "constructivism," "mathematical empowerment," "facilitator").
- Those who fear that early use of calculators will lead to further erosion in basic skills.
- Those who believe that mathematically gifted children need to be grouped with their peers.
- Those who see in low test scores evidence of failed education.
- Those who believe that teachers should teach, not coach, and that students should learn, not explore.
- Those who believe that public-supported education is responsible for providing occupational skills.
- Those who demand hard evidence of likely success before risking children in an educational experiment.
- Those who urge greater emphasis on computers to prepare students for a high-performance, technical workplace.
- Those who value mathematics as they learned it.
- Those who view mathematics as a training ground for virtues of discipline, orderliness, logic, and accuracy.

One could undoubtedly find more complaints, but these are sufficient to make my main point: NCTM's view is not the only one at the owner's table of school mathematics. It may not even be the majority view.
Four Perspectives

One can analyze these concerns from many perspectives. I choose four important constituencies that reveal the diversity of views:

• Two from the public--parents and employers;
• Two from education--professors and vocational educators.

Parents, of course, view themselves as the primary "owners" of schools, as they should. They tend to believe that schools should reflect the traditions and values of the community they serve, which usually are not the same as the values of educators, or of NCTM. All too often, their values are those of a glamorized past--school as they imagine it to have been when they grew up.

Here are some of their beliefs and concerns:

• Basic computational skills are the foundation of excellence in mathematics.
• Standard textbooks are essential for quality, consistency, and parental involvement.
• The primary emphasis in mathematics class should be on math skills, not on writing.
• In mathematics, correct methods and right answers are of paramount importance.
• Early regular use of calculators is unwise.
• Teachers cannot meet all children's needs in mixed-ability classrooms.
• Some children want and need more rigorous mathematics than others.
• Mixed ability groups retard the learning of advanced students.
• Teachers should be instructors, not "facilitators."
• Group problem solving comes at the expense of individual performance.
• Innovative curricula unfairly make children into guinea pigs in educational experiments.

Professors, as one would expect, hold a variety of views. Some are strong advocates, others strong critics. Most are still in the dark. Both advocates and critics agree on certain strengths. For example, all welcome the inclusion of probability and statistics, and virtually all agree that concrete problems galvanize student interest.

But critics in colleges and universities have raised forceful objections:

• Reduced attention to drill undermines the technical fluency required for success in subsequent courses, especially in science and mathematics.
• Standards-based programs fail to challenge the most capable children to a high level.
• Many topics important for higher mathematics are not emphasized sufficiently (e.g., quadratic equations, geometric series, proofs).
• Exploration and discovery comes at the expense of rigor and correct answers.
• In the name of relevance, abstraction has declined as a motivation for mathematics.
• Teachers provide too little summary of key formulas that students must learn.
• Calculation and proof--the hallmarks of mathematics--receive insufficient emphasis.
• Group activities are overemphasized as compared with individual accomplishment.

Employers express surprisingly different priorities--different from educators, and different from each other. In the emerging occupational skills standards, mathematics often appears as a thin set of basic skills--a caricature of the "back-to-basics" movement. Here, for example, are some
of the mathematical skills stipulated by the new occupational standards for chemical technicians:

- Calculate percentages and ratios
- Read and construct graphs using different scales
- Solve simple algebraic equations
- Recognize patterns from data
- Calculate and understand standard deviation
- Describe accuracy and precision
- Develop and interpret control charts
- Perform unit conversions
- Evaluate propagation of error
- Use differentials to calculate rates

In sharp contrast, the influential 1991 report "What Work Requires of Schools" from the [Labor] Secretary's Commission on Achieving Necessary Skills (SCANS) asks not for percentages, ratios, and trigonometry, but for five broad competencies built on a foundation of basic skills, thinking skills, and personal qualities:

**Competencies:**

- **Resources:** Managing time, money, material, facilities, and human resources.
- **Interpersonal:** Capacity for teamwork, teaching, service, leadership, negotiation, diversity.
- **Information:** Ability to acquire, evaluate, organize, maintain, interpret, communicate, and transform information.
- **Systems:** Understanding, monitoring, and improving social, organizational, and technological systems.
- **Technology:** Selecting, applying, and maintaining technology.

**Foundation:**

- **Basic Skills:** Reading, writing, listening, speaking, arithmetic/mathematics.
- **Thinking Skills:** Creative thinking, reasoning, problem solving, decision-making, processing symbols, acquiring and applying new knowledge.
- **Personal Qualities:** Responsibility, self-esteem, sociability, self-management, integrity.

Finally, polls of employers tell yet another story: what employers want most from employees is diligence, reliability, and other virtues of the American work-ethic:

- To take initiative and accept responsibility for one’s work;
- To cooperate with others and work in groups;
- To plan and evaluate one’s own work and the work of others;
- To work with persons of different backgrounds and cultures;
- To make informed decisions based on careful investigation;
- To identify the resources necessary to solve a problem;
- To continue learning technically sophisticated ideas and skills.
Vocational education, the fourth of my chosen constituencies, represents the most politically powerful trend in school reform these days. Here are just a few examples to illustrate this high-level interest:

- Wisconsin Governor Tommy Thompson, 1995-96 chair of the Education Commission of the States, has made "Connecting Learning and Work" the major theme of his term as Chair.
- The Association of American Colleges and Universities (AAC&U), an organization devoted to liberal education, has selected as the theme of its forthcoming annual meeting "Work & Learning: Creating New Connections."
- Later this month, in San Francisco, there is a major conference on "The Education of the new California Workforce."
- At the end of March, the National Governors' Association is planning a summit on education whose focus will be on employment and technology.

Vocational education has suffered greatly from its image as a second class track—a program for someone else's children. Parents criticize traditional vocational education for failing to ensure employment. Business leaders complain that these programs provide insufficient academic skills. University leaders routinely dismiss "applied courses" as unworthy of credit toward admission or transfer. Very little has gone right for traditional "voc-ed."

Our nation needs better articulation from school to work, to reduce the problem of "floundering youth" who now take more than a decade after leaving school to find suitable, stable work. Fortunately, a new consensus is emerging for integration of vocational and academic programs based on concrete education in authentic contexts:

- All students deserve an education that prepares them for work and for further study.
- Vocational curricula should be just as demanding intellectually as traditional academic programs.
- Transitions from school to work need to be more transparent to students, more effective for employers, and more suited to a highly mobile labor force.
- Learning in specific contexts (e.g., apprenticeships, work-study, and independent projects) provides powerful educational leverage.
- Broad industry-wide emphases provide sounder education than narrow skills for specific trades.
- Educators and employers need to coordinate their different sets of standards.

Mathematics in the workplace makes sophisticated use of elementary mathematics rather than elementary use of sophisticated mathematics (such as that studied in the final years of high school). Technical employees are frequently called on to create multi-step solutions to open-ended problems using relatively low-level mathematical tools. Effective solutions require a high degree of accuracy within required tolerances. To achieve the desired degree of reliability, problems must routinely be solved in different ways to check results. All this calls for very sophisticated problem-solving strategies but not often for very sophisticated mathematics.
The essential tools for workplace mathematics are those of concrete mathematics:

- Sophisticated computation
- Three-dimensional geometry
- Indirect measurement
- Statistical Quality Control
- Graphical representation
- Calculators & computers
- Measuring devices
- Trigonometry
- Measurement
- Calculation
- Difference Equations
  - Data analysis
- Spreadsheets
- CAD/CAM

Concrete mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. In the workplace,

- Geometry is used not so much to prove results as for modeling and measurement, primarily in three dimensions.
- Algebra is used not so much to solve equations but to represent complex relationships in symbolic form.
- Numbers are used not just to represent quantities, but also to calculate tolerances and limit errors.

Concrete mathematics emphasizes numbers in context--numbers that are used with appropriate units of measurement, supported by computer graphics for visualization, and embedded in authentic models.

**Concluding Observations**

I opened by comparing the fragmented claims of ownership of school mathematics to the fractious and schism-inclined history of religions. This comparison served, perhaps, to illustrate the wide diversity in perspectives of various constituents of mathematics education. Yet apart from the many wars that have ensued from religious differences, variety in religious options is generally a good thing.

In contrast, the sharply divergent views of mathematics education can have profoundly negative consequences for our students: The tightening of the economy has shifted the public interest in education from enlightenment to employability, yet educators are still mostly interested in the former without much regard to the latter. It is students who suffer from this schism. Employers do not regard school records--neither grades, nor teacher recommendations, nor test scores--as good indicators of employability. Again, it is students who suffer as a result. Higher education faculty are not much impressed by transfer of credits, only by the ability to perform in subsequent courses. Students thus often lose their good-faith investments in course credits. The public is more interested in accountability of educational expenditures than in education of students.

Our educational system is--to shift metaphors--like an Alice-in Wonderland transportation industry, in which the Red Queen makes engines designed for automobiles that are delivered to the Mad Hatter for installation in railway cars that he is making to fill an order for airplanes.
The *Standards* are a wonderful illustration of thoughtful automotive engineering, but have been delivered to an educational system that is busy making railroad cars for a public that expects airplanes. It is no wonder that there is some confusion abroad in the land.

I conclude with some recommendations for reflection and reconciliation:

- Listen to our constituents as a sign of respect and humility.
- Educate everyone equally not identically, but comparably.
- Emphasize concrete mathematics as a stepping-stone to abstraction.
- Prepare students for work and also for further education.
- Preserve mathematical integrity via a strong core curriculum.
- Collaborate with others to place mathematics in context.
- Employ computer networking for experience in using modern tools.
- Provide evidence of accomplishment for public accountability.

**Further Reading**

