Finding derivative functions for power functions

We know that the slope of the tangent line to the graph of a function f(x) at x=a is found by the following formula:

$$\lim_{h ?\ 0} \frac{f(a ? h) ? f(a)}{h}$$

1. Explain what each of these parts of the formula mean in terms of the graph of a function f(x). Draw a picture if you wish.

a. $\frac{f(a ? h) ? f(a)}{h}$

b. $\lim_{h ?\ 0} \frac{f(a ? h) ? f(a)}{h}$

2. Now, we will use the same idea to find a general formula for the slope of the tangent line to the graph of a function at any point. This general formula gives us a function of x, called the derivative function of f(x). Since we want a formula that is true for any x, we replace a in the formula above with x.

$$f'(x) ? \lim_{h ?\ 0} \frac{f(x ? h) ? f(x)}{h}$$

3. To see how this works, we find the derivative function for f(x)=x^2, using the above formula. Fill in the missing reasons:

a. $f(x) ? x^2$ — Function

b. $f'(x) ? \lim_{h ?\ 0} \frac{f(x ? h) ? f(x)}{h}$ — Definition of derivative function

c. $f'(x) ? \lim_{h ?\ 0} \frac{(x ? h)^2 ? x^2}{h}$ — Replace the generic f(x) with the actual function

d. $f'(x) ? \lim_{h ?\ 0} \frac{x^2 ? 2xh ? h^2 ? x^2}{h}$ — Expand f(x+h)

e. $f'(x) ? \lim_{h ?\ 0} \frac{2xh ? h^2}{h}$ — ______________________________

f. $f'(x) ? \lim_{h ?\ 0} \frac{h(2x ? h)}{h}$ — ______________________________

g. $f'(x) ? \lim_{h ?\ 0} 2x ? h$ — ______________________________

h. $f'(x) ? 2x$ — ______________________________

4. Your goal is to come up with a general formula for the derivative function of $f(x) = x^n$ for any value of n. First use the TI-89 to help you expand f(x+h) for several functions $f(x)=x^n$ where x varies. Use F2 -> 3. expand to get expand, then compute expand ((x+h)^n,x)| n=3. (The | key is 2nd k.) Just change the value of n to expand other power functions.

$f(x)$	Expanded form of $f(x ? h)$	$f(x)$	First five terms of Expanded form of $f(x ? h)$
x^3		x^{15}	
x^4		x^{20}	
x^5		x^{30}	

Now put away your calculator and do the rest of this task by hand, using the results from question 4.

5. For each of the functions in the table in question 4, compute the derivative function, completing steps d through h as listed in question 3.

$f(x)$	x^2	x^3	x^4	x^5
d. $f'(x) =$	$\lim_{h ? 0} \frac{x^2 ? 2xh ? h^2 ? x^2}{h}$			
e. $f'(x) =$	$\lim_{h ? 0} \frac{2xh ? h^2}{h}$			
f. $f'(x) =$	$\lim_{h ? 0} \frac{h(2x ? h)}{h}$			
g. $f'(x) =$	$\lim_{h ? 0} 2x ? h$			
h. $f'(x) =$	$2x$			

Continue with additional functions from question 4 until you can predict the derivative of f(x)=x^n for any n.

5. If f(x) = x^n, what is the formula for f'(x)?

6. What happens between step d and step e with each of the functions you tried?

7. Why don't you need all of the terms of the expanded form of f(x+h) to find the results?

8. How can you predict derivative by just looking at the expanded form of f(x+h)?