M128A: Slope Fields by Numbers

Due Wednesday, February 26, 2003

1. To recall (or learn for the first time) how to graph a slope file on your calculator, use the Slope Fields directions to graph a slope field for the differential equation $f'(x) = 2x$. However, there is a mistake in the directions: in step (ii) at the top of the page, you should enter y1'=2t, not y1'=2. (Note 1: differential equation mode uses t as the independent variable, not x.) (Note 2: a corrected version of the Slope Field directions is accessible from syllabus) .

2. Try it again, only this time, don't use F8 to set the initial value. Instead, in the Y= editor, set yi1=0. This says that $y(0) = 0$. After you graph, use the trace key to trace the y function that results from your initial value problem: $f'(x) = 2x, f(0) = 0$. What is the value of $f(1.2)$ for this solution function?

3. On the half-sheet slope field that I handed out, draw the solution for the initial value problem $f'(x) = x - f(x), \quad f(0) = 1$ which is the same as $y' = t - y, \quad y(0) = 1$. (Note: the horizontal axis is the t-axis and the vertical axis is the y-axis.) Remember, that you put a mark at the initial point on the graph, then try to follow the slope marks to construct the function. In the solution function, what do you think is the y-value when $t = -1$?

4. Now try $f'(x) = x - f(x)$. You would enter this in your calculator as y1'= t - y1. Fiddle with the window to try to get a picture on your calculator that looks like the little sheet that I handed out. (Hint, adjust the x and y mins and maxes – you may have to adjust the t max as well.) Now, try starting at different spots, first using the F8 key as described on your sheet, then setting y1i to some initial number.

5. Use the calculator graph in (4) above and the trace key to predict what $f(-1)$ would be for the above differential equation if we know that f(0) = 1. (Hint: you can only use the trace key if you put the initial value in by setting $y1i$ = the initial value, not by using the F8 key.

   $f'(x) = x - f(x), \quad f(0) = 1$ which is the same as $y' = t - y, \quad y(0) = 1$

6. Now, go back to the initial value problem in question (3) $f'(x) = 2x, f(0) = 0$. Our goal is to estimate the value of $f(1.2)$ numerically instead of graphically. Fill in the missing values below, calculated only from the differential equation and the initial value (not by looking at the graph on the calculator).

   a) $f(0) =$ ______

b) $f'(0) = 0$. Why is this true?

c) Assume that the **slope** of f remains constant (i.e. $f'(x) = 0$) in the interval $[0, .2)$. What would then be the value of $f(.2)$? ______

d) $f'(.2) =$ ______

e) Again assume that the **slope** of f remains constant in the interval $[.2, .4)$. What would then be the value of $f(.4)$?

f) Continue, the process above, calculating $f(x$ and $f'(x)$ at intervals of .2 until you get to 1.2. What is the value of $f(1.2)$?

g) Go through the whole process again, using intervals of .1 instead of .2. Which subinterval size gives you an estimated value $f(1.2)$ that is closer to the calculator value? Why?

6. Now try the same process with the initial value problem in part (4): $f'(x) = x - f(x), \quad f(0) = 1$. Estimate $f(1)$ in using subintervals of length .2. Here are the beginning steps:

   a) $f(0) = 1$. Why?

   b) $f'(0) = 0 - 1 = -1$. Why?

   c) Assume that the **slope** of f remains constant (i.e. $f'(x) = -1$) in the interval $[0, .2)$. Then $f(.2) = 1 + (-1) * .2 = .8$. Draw a picture to convince yourself of this fact.

   d) $f'(.2) = .2 - .8 = -.6$. Why?

   e) Again assume that the **slope** of f remains constant in the interval $[.2, .4)$. Then, $f(.4) = .8 + (-.6) * .2$. Why?

   f) Continue this process to find an estimate for $f(1)$.

7. Finally try the same process with the same initial value problem as in part (6), only change your subintervals to be of length .1. Compare both of your estimates for $f(1)$ to one that you can get by tracing on the calculator. Which gives you a closer estimate for $f(1)$?

8. See if you can come up with an algorithm for any subinterval length, say delta t. You are starting at t=0 and wanting to estimate f(1). How could you do it?

Be sure to have this done as much as you are able by Wednesday's class.