Part 2 · Promoting Effective Classroom Instruction and Assessment

"Learning mathematics requires construction, not passive reception, and to know mathematics requires constructive work with mathematics objects in a mathematical community."

Davis, Maher, & Noddings, 1990

" No other decision that teachers make has greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics. Here the teacher is the architect, the designer of the curriculum."

Lappan, 1993

Planning Meaningful Tasks

Background

Choosing meaningful learning experiences for students is one of the most important instructional decisions a teacher makes. The importance of worthwhile tasks is addressed in the *Professional Standards for Teaching Mathematics*. "The mathematics tasks in which students engage—projects, problems, constructions, applications, exercises, and so on—and the materials with which they work frame and focus students' opportunities for learning mathematics in school" (NCTM, 1991, p. 24).

Reys and Long (1995, p. 297) characterize good tasks as those that:

- are often authentic, coming from the students' environment
- are challenging, yet within reach of students
- pique and take advantage of students' curiosity
- encourage multiple perspectives and interrelated mathematical ideas
- nest skill development in the context of problem solving
- encourage students to make sense of mathematical ideas

What Research Says

" Students will not be provoked to inquire, learn, or study what they already know, or think they know, or what they consider at the moment to be irrelevant."

Rowe, 1978

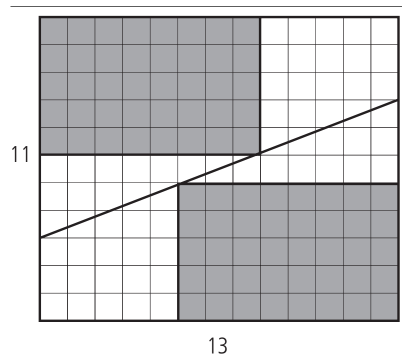
Support for problem-centered instruction can be found in a number of research studies including the work of Wood and Sellers (1997), who compared achievement of students receiving zero to two years of problem-centered instruction. Those in the two year group had "significantly higher scores on standardized achievement measures, better conceptual understanding, and more task-oriented beliefs for learning mathematics" (p. 163). Teachers can support student construction of mathematical understanding by embedding worthwhile tasks in a series of connected problems. Mathematics learning is enhanced when tasks provide students opportunities to use previously learned concepts and techniques in the process of discovering new ones (Lappan and Briars, 1995).

Choosing meaningful tasks involves two important components: the role of context in learning and the role of conceptual conflict in accommodating new knowledge (Rowe, 1978, p. 180). Context represents the situations in which activities are embedded. These become part of what is learned and how that learning is remembered and recalled (Lappan & Briars, 1995). Students who comment, "This is the same math we used to solve the Ferris Wheel problem!" are relating the mathematics to the context in which it was learned.

" The teacher needs to change the emphasis of problem solving from "Here's a problem, solve it" to "Here's a situation, let's explore it!"

Showalter, 1994

Conceptual conflict exists when new information does not seem to fit with what one already knows. When teachers produce situations in which there exists room for differences of interpretation, students will attempt to resolve those differences. The following example, An Area Paradox (Jacobs, 1970, p. 21), can create conceptual conflict for students which motivates them to investigate the mathematics in order to resolve the situation.



EX: Cut out the six pieces in the figure and throw the two shaded rectangular pieces away. The area of the remaining pieces is 63 square units. Now rearrange the four pieces to form a square. What is the area of this figure? Explain how this can be.

Implications for the Classroom

Designing or choosing tasks demands thoughtful and often collaborative planning on the teacher's part. A good mathematical investigation is one that not only invites finding one or more solutions but also allows for extensions beyond the immediate problem situation. Teachers can help facilitate such possibilities by scaffolding questions that allow students to continue reasoning through the problem or by asking challenging questions that motivate students to pursue problem extensions.

"The tasks in which students engage must encourage them to reason about mathematical ideas, to make connections, and to formulate, grapple with, and solve problems."
NCTM, 1991

The following list (Lappan et al., 1996, p. 40) can be used as a guide for evaluating classroom activities and reexamining what students are being asked to do in their mathematics textbooks. A rich problem solving task:

- has important, useful mathematics embedded in it
- may have different solutions or allow for different decisions or positions to be taken and defended
- can be approached by students in multiple ways using different solution strategies
- encourages student engagement and discourse
- requires higher level thinking and problem solving
- contributes to the conceptual development of students
- promotes the skillful use of mathematics
- creates opportunities for teachers to assess what their students are learning and where they are having difficulty

Engagement is necessary but not sufficient for mathematical learning. Student discussion and reflection must accompany active engagement to develop understanding of mathematical concepts. Also critical to concept development are the challenges inherent in the task, the materials and technology used to support the investigation, and student interaction focused on making sense of the mathematics. While students are engaged in problem-solving tasks, the teacher must listen to students' conversations and ask key questions. This provides the teacher with insights into levels of student understanding and further instructional needs.

For Further Study

Armstrong, B. (1995). "Teaching patterns, relationships, and multiplication as worthwhile mathematical tasks." *Teaching Children Mathematics*, 1(7), 446-450.

Showalter, M. (1994). "Using problems to implement the NCTM's professional teaching standards." *Mathematics Teacher*, 84(1), 5-7.

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Using Concrete Materials

Background

"The child invents mathematical knowledge from her or his actions on objects, so direct, concrete experiences with many objects at the child's developmental level are crucial to the formation of accurate concepts."
Maxim, 1989

The NCTM *Professional Standards for Teaching Mathematics* recommend the use of concrete materials as one of many tools for enhancing communication and mathematical reasoning. Success with manipulatives depends greatly on the teacher's ability to choose appropriate objects and to create connections with the underlying mathematical concepts. Students do not always associate their work with concrete materials to the corresponding mathematical abstractions. Teachers can help students complete the process by asking probing questions and encouraging students to make these connections explicit.

When used wisely, manipulatives can:

- help make abstract ideas concrete
- lift mathematics off textbook pages
- help students construct connections between mathematical ideas, vocabulary, and symbols
- give students physical evidence to test and confirm their reasoning
- serve as concrete models for students to use to solve problems
- intrigue and motivate while helping students learn (Burns, 1996, p. 47)

What Research Says

According to research, explorations and investigations with manipulatives are an excellent way of providing students with tangible mathematical experience. Using manipulatives promotes active learning, models mathematics, and builds motivation. Studies indicate that the use of concrete objects is integrally related to the development of meaning. As children work with objects and talk about what they are doing, meaning is created and assimilated.

"Students who learn with manipulatives are better able to cross the bridge to the abstract world of mathematical concepts and apply their knowledge to real-life situations."
Kober, 1991

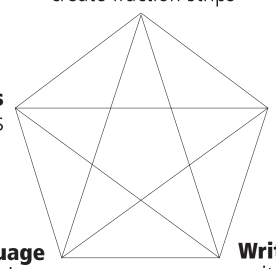
A review by Suydam and Higgins (Suydam, 1982) concerning manipulative use found that:

- lessons using manipulative materials are more likely to produce greater mathematical achievement
- the inclusion of the concrete stage in a sequence of instruction improves achievement
- studies at every grade level and across a variety of mathematical topics support the importance of the use of manipulative materials
- the use of materials appears to be effective with children at all achievement levels and all ability levels

Sowell (1989) concluded that long-term use of concrete materials increased student achievement more than short-term use. In addition, teacher training and knowledge of manipulative use was a key factor in influencing the effectiveness of instruction.

The research of Piaget, Dienes, and Bruner suggests that children's concepts evolve from direct interaction with the environment. Clements and McMillen (1996) capture this when they write, "Mathematical ideas are ultimately...integrated...not by their physical or real-world characteristics but rather by how 'meaningful'—connected to other ideas and situations—they are" (p. 271). Lesh (1979) found that using manipulatives in conjunction with pictorial, verbal, symbolic, and real-world representations can maximize learning. Connections to, and translations between, different representations are important cognitive processes which lead to a more robust understanding of concepts. Figure 1 shows the Lesh model applied to various representations of fractions.

Figure 1



Implications for the classroom

Effective instruction integrates experiences with concrete materials, which should be used:

- frequently
- primarily by students rather than by teachers for demonstration purposes
- in conjunction with other tools and conceptual representations
- in ways appropriate to the mathematical content under investigation
- in conjunction with exploratory and inductive instructional approaches
- as an aid in organizing content

"...allowing students to move the objects themselves is preferable to having a teacher demonstrate the action."
Kober, 1991

The scope of manipulative use in classrooms needs to be expanded. Tools such as computers, calculators, and measuring and drawing devices should complement the more familiar materials including geoboards, base ten blocks, and algebra tiles. Children need opportunities to use objects appropriate to the mathematical concepts being studied, the children's developmental levels, and their learning styles. Instruction should encourage children to use flexible and informal methods and avoid rote manipulation of materials.

"Teachers and students should avoid using manipulatives as an end—without careful thought—rather than as a means to that end."
Clements & McMillen, 1996

Not all types of manipulatives are right for all children, nor is one set of manipulatives appropriate for teaching all topics. However, worthwhile lessons, activities, or units that effectively use concrete materials engage both students' hands and minds. Careful use and sequencing of tasks involving manipulatives can help students create a bridge between concrete models and operations or abstract concepts. This leads to a deeper understanding of mathematics and requires students to develop evidence-based explanations for how things work.

For Further Study

Ball, D.L. (1992). "Magical hopes: Manipulatives and the reform of math education." *American Educator*, 16(2), 14-18, 46-47.

Clements, D. & McMillen, S. (1996) "Rethinking 'concrete manipulatives.'" *Teaching Children Mathematics*, 2(5), 270-279.

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Integrating Technology

Background

Life in the 21st century will require personal competence with technology, information processing, communication, and decision making. With the integration of technology in every aspect of the workplace, the integration of technology in teaching and learning is essential. New, easy-to-use technologies offer increasingly versatile tools that can supplement and reinforce, not replace, student learning.

"Increased use of technology in mathematics education is inevitable, but wise use is not automatic. Effective use of calculators and computers requires objectives for mathematics education that are aligned with the mathematical needs of the information age."
MSEB, 1990

Used appropriately, new technology can enhance opportunities for children to engage in higher-order thinking. For example, the TI-92 graphing calculator and *Mathematica* and *Maple* software can perform the routine manipulation of symbols typically associated with the study of high school mathematics from algebra through calculus allowing students to focus on deeper structural elements of mathematics. *Geometer's Sketchpad* and *Cabri Geometry* are part of the growing number of dynamic geometry software programs that allow students to actively explore the interplay between shape, space, and measurement. Students can represent and manipulate geometric figures, investigate relationships among figures, and explore and test geometric conjectures. The Internet continues to provide new instructional and exploratory environments through interactive use of "notebooks," computer files specifically developed for mathematical explorations.

"Failure to introduce and to use calculators and computers in school creates a needless barrier between what is happening in students' everyday lives and what they are being taught in school."
Shane & Tabler, 1981

Swadener & Blubaugh (1990) believe that technology must be fully available to all students and teachers at all times as part of an instructional emphasis on problem solving skills and concept development. An NCTM position statement (1994) recommends that, "Teachers should use computers as tools to assist students with the exploration and discovery of concepts, with the transition from concrete experiences to abstract mathematical ideas, with the practice of skills, and with the process of problem solving."

Students who have not learned to use a calculator effectively to explore problems will be at a disadvantage in many testing situations. The Scholastic Aptitude Test (SAT) and American College Test (ACT) now endorse calculator usage while the Advanced Placement (AP) Calculus and Statistics tests are written so that calculator usage is necessary or advantageous.

What Research Says

Research results indicate that calculator use for instruction and testing enhances learning and performance. This is true for arithmetical concepts and skills as well as problem solving (Hembree & Dessart, 1992). Research is consistent in showing that calculators do not have a negative impact on students' computational skills, including mastery of basic facts and proficiency in applying computational algorithms (Hembree & Dessart, 1986).

"The major influence of technology...is its potential to shift from an emphasis on skills to an emphasis on developing concepts, relationships, structures, and problem-solving."
Corbitt, 1985

It is now clear that an understanding of arithmetic can be developed with a curriculum that emphasizes estimation, mental arithmetic, and calculator use, with reduced instruction in paper and pencil calculation. Indeed, there is evidence that overemphasis on manual skills hinders children's learning of when and how to use those skills (MSEB, 1990).

Researchers have also studied the effects of computer graphics on student understanding. Early studies of *Logo* and *Geometric Supposer* software suggest that students using exploratory programs may perform as well as or better than other students on traditional criteria (MSEB, 1990). For mathematical concepts, such as data analysis or functions, the computer seems to enhance student interest and understanding of important ideas. In addition, Demana and Waits (1990, p. 212) suggest that the greatest benefits appear to come from interactive technology that:

- is controlled by the user
- promotes student exploration
- enables generalization

Implications for the Classroom

Technology impacts the mathematics curriculum in three major ways (NCTM 1991):

- some mathematics becomes more important because technology requires it
- some mathematics becomes less important because technology replaces it
- some mathematics becomes possible because technology allows it

"Rather than replacing pencil and paper computation procedures, calculators are more likely to reinforce them; rather than substituting for independent thought, calculators are apt to sustain it."
Kober, 1991

Since computers and calculators can be invaluable tools to help students focus on generating ideas, trying out various approaches, and checking hunches, technology can help make mathematics accessible. If the context and challenge of a problem can be grasped by the students, then computation is no longer a barrier to solving the problem. The opportunity to use technology in doing mathematics allows students to:

- explore very complex problems and sophisticated concepts, with real data from real experiments and applications
- focus on strategies and test predictions with greater ease
- discuss and reflect on the mathematical principles behind the operations
- estimate, predict, share their discoveries, and question the reasonableness of their answers
- decide what type of technology tool/method is appropriate in a given situation
- learn the limitations of calculators, computers, and other technological tools

The unique instructional opportunities provided by calculators and computers support a change in teaching emphasis from getting the answer to reflecting on the process. No one today knows just what technology applications will be available in the next year or the next century. Instruction that emphasizes familiarity and flexibility with technological tools, conceptual understanding of mathematics, and facility with problem-solving approaches will be necessary to better equip students to deal with their future in an increasingly technological world.

For Further Study

Campbell, P.F. & Stewart, E.L. (1993). "Calculators and computers." In R.J. Jensen (Ed.), *Research ideas for the classroom: Early childhood mathematics*. Reston, VA: NCTM.

Fey, J.T. & Hirsch, C.R. (1992). *Calculators in mathematics education: 1992 yearbook*. Reston, VA: NCTM.

King, J. & Schattschneider, D. (Eds.). (1997). *Geometry turned on: Dynamic software in learning, teaching and research*. Washington, D.C.: The Mathematics Association of America.

"What we assess and how we assess it communicates what we value."
NCTM, 1995

"Assessment involves gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and making inferences from that evidence for a variety of purposes."
NCTM, 1995

"Prior knowledge, experience, and the opportunity to learn are important considerations in interpreting test results."
NCTM, 1989

Assessing Student Performance

Background

Assessment is the process of describing what mathematics students know and can do. Some think that assessment, evaluation, and testing are synonymous. They are not.

Assessment is the process of gathering information. Once data have been collected, the diverse pieces of information are interpreted and integrated into a summary judgment. This is *evaluation*. *Tests* are measuring devices that are used to document student learning on narrowly defined questions and tasks. They are intended to produce a score in a more-or-less neutral and decontextualized environment. The tendency in the past has been to reduce assessment to testing, often using the data from a single test to indicate a student's capabilities. In reality, tests are just one of many tools available to the assessment process.

Because assessment is a feedback mechanism, it communicates teachers' expectations for students. For example, if communicating and defending a solution strategy to a mathematics problem is important, assessment should reflect this. On the other hand, when a student's knowledge of specific facts or algorithms is required, a multiple-choice or short answer test may be appropriate. Assessments must reflect the learning goals we have for our students as well as our instructional strategies.

What Research Says

Different aspects of achievement are measured by traditional tests and performance measures. For example, research indicates that students do not necessarily do better on performance tests than on other tests; they perform differently because the tasks are different. There is often a wide gap between the ability of students to demonstrate procedures and their ability to explain the procedures.

For these reasons, a number of meaningful tasks are required to be able to generalize about student performance and/or program effectiveness. Additionally, a variety of assessment formats are required to obtain a comprehensive view of student achievement. It is important to realize that performance tests are not equally interchangeable and each appears to measure different yet related aspects of mathematics achievement.

Questions of fairness and equity are as important in performance assessments as in other forms of assessment. Wording, topic or task selection, format, and scoring approaches can all introduce bias and influence performance.

Implications for the Classroom

Good assessment in mathematics:

- focuses on what students *know* and *can do* rather than what they *do not know*
- matches the curriculum in both *what is taught* and *how it is taught*
- is unbiased and fair for all students
- allows students to learn at various paces and honors various styles of learning and performance
- uses questions that require thoughtful responses
- compares performance over time to help recognize patterns of success and/or difficulty

Recent techniques collectively known as "alternative," "authentic," "active," and "performance" assessments reflect new understandings of how students learn. These assessments encompass a variety of methods, including: enhanced multiple-choice, teacher observations, checklists, investigations, portfolios, computer simulations, projects, and student self-assessments. Formats which allow for alternative ways to demonstrate understanding provide for individual differences. These differences may be the result of culture, gender, primary language, economic background, disabilities, or other factors that are masked in a single, traditional form of assessment.

" Assessments can contribute to students' opportunities to learn important mathematics only if they reflect, and are reinforced by, high expectations for every student."
MSEB, 1993

Rich learning activities can provide opportunities for informal assessment and can be modified to become performance assessment tasks. Professional educational journals and books also include examples and descriptions of teachers' experiences with authentic assessment. However, even with these resources, the process of developing assessments is often time-consuming and sometimes frustrating. Stenmark (1991, p. 1) makes the following recommendations for educators making changes in classroom assessment practice:

- don't try to do it all at once; start small but start somewhere
- don't try to do it all alone; find someone to work with, preferably at your grade level or within your discipline

The first step in developing assessment tasks is to identify the "big idea" of a unit. This is the "What is worth knowing?" question. Next, the criteria for judging student learning should be developed. Checklists are a useful way of displaying criteria and of clearly identifying the characteristics of a good product. Finally, students should have access to the checklists as they prepare for and complete an assessment. They can also use the checklists to make a judgment about the quality of their work that can be used as the basis for a discussion with them about their progress.

" In order to develop mathematical power in all students, assessment needs to support the continued mathematics learning of each student."
NCTM, 1995

Expanding the scope of assessment provides a broader view of student capabilities and knowledge. Teachers are then better prepared to describe and comment on each student's learning to parents and administrators, as well as to the students themselves. In addition, continual assessment of student understanding helps guide both long and short range instructional decisions designed to:

- ensure that every student is learning sound and significant mathematics
- support the development of a positive disposition toward mathematics
- challenge and extend student's ideas
- identify student needs in order to adapt or change activities

There is a dynamic interplay between instruction and assessment. Ideally, the lines between instruction and assessment become blurred when tests are part of instruction and instructional tasks are rich diagnostic opportunities.

For Further Study

Mathematical Sciences Education Board (MSEB). (1993). *Measuring what counts: A conceptual guide for mathematics assessment*. Washington, D.C.: National Academy Press.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

Stenmark, J. (Ed.). (1991). *Mathematics assessment: Myths, models, good questions, and practical suggestions*. Reston, VA: NCTM.