Minnesota K-12 Academic Standards in Mathematics

April 14, 2007 Revision

Grades 5-11

Sorted by Standard:

1. Number & Operation
2. Algebra
3. Geometry and Measurement
4. Data Analysis

5	Algebra	Recognize and represent patterns of change; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	5.2.1.1	Create and use rules, tables, spreadsheets and graphs to describe patterns of change and solve problems. *For example*: An end-of-the-year party for 5th grade costs $100 to rent the room and $4.50 for each student. Know how to use a spreadsheet to create an input-output table that records the total cost of the party for any number of students between 90 and 150.
			5.2.1.2	Use a rule or table to represent ordered pairs of positive integers and graph these ordered pairs on a coordinate system.
		Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving whole numbers.	5.2.2.1	Apply the commutative, associative and distributive properties and order of operations to generate equivalent numerical expressions and to solve problems involving whole numbers. *For example*: Purchase 5 pencils at 19 cents and 7 erasers at 19 cents. The numerical expression is $5 \times 19 + 7 \times 19$ which is the same as $(5 + 7) \times 19$.
		Understand and interpret equations and inequalities involving variables and whole numbers, and use them to represent and solve real-world and mathematical problems.	5.2.3.1	Determine whether an equation or inequality involving a variable is true or false for a given value of the variable. *For example*: Determine whether the inequality $1.5 + x < 10$ is true for $x = 2.8$, $x = 8.1$, or $x = 9.2$.
			5.2.3.2	Represent real-world situations using equations and inequalities involving variables. Create real-world situations corresponding to equations and inequalities. *For example*: $250 – 27 \times a = b$ can be used to represent the number of sheets of paper remaining from a packet of 250 when each student in a class of 27 is given a certain number of sheets.
			5.2.3.3	Evaluate expressions and solve equations involving variables when values for the variables are given. *For example*: Using the formula, $A = \ell w$, determine the area when the length is 5, and the width 6, and find the length when the area is 24 and the width is 4.

6	Algebra	Recognize and represent relationships between varying quantities; translate from one representation to another; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	6.2.1.1	Understand that a variable can be used to represent a quantity that can change, often in relationship to another changing quantity. Use variables in various contexts. *For example*: If a student earns $7 an hour in a job, the amount of money earned can be represented by a variable and is related to the number of hours worked, which also can be represented by a variable.
			6.2.1.2	Represent the relationship between two varying quantities with function rules, graphs and tables; translate between any two of these representations. *For example*: Describe the terms in the sequence of perfect squares $t = 1, 4, 9, 16, \ldots$ by using the rule $t = n^2$ for $n = 1, 2, 3, 4, \ldots$.
		Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving positive rational numbers.	6.2.2.1	Apply the associative, commutative and distributive properties and order of operations to generate equivalent expressions and to solve problems involving positive rational numbers. *For example*: $\frac{32}{15}\times\frac{5}{6}=\frac{32\times5}{15\times6}=\frac{2\times16\times5}{3\times5\times3\times2}=\frac{16}{9}\times\frac{2}{2}\times\frac{5}{5}=\frac{16}{9}$. *Another example*: Use the distributive law to write: $\frac{1}{2}+\frac{1}{3}\left(\frac{9}{2}-\frac{15}{8}\right)=\frac{1}{2}+\frac{1}{3}\times\frac{9}{2}-\frac{1}{3}\times\frac{15}{8}=\frac{1}{2}+\frac{3}{2}-\frac{5}{8}=2-\frac{5}{8}=1\frac{3}{8}$.
		Understand and interpret equations and inequalities involving variables and positive rational numbers. Use equations and inequalities to represent real-world and mathematical problems; use the idea of maintaining equality to solve equations. Interpret solutions in the original context.	6.2.3.1	Represent real-world or mathematical situations using equations and inequalities involving variables and positive rational numbers. *For example*: The number of miles m in a k kilometer race is represented by the equation $m = 0.62\,k$.
			6.2.3.2	Solve equations involving positive rational numbers using number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation. Interpret a solution in the original context and assess the reasonableness of results. *For example*: A cellular phone company charges $0.12 per minute. If the bill was $11.40 in April, how many minutes were used?

7	Algebra	Understand the concept of proportionality in real-world and mathematical situations, and distinguish between proportional and other relationships.	7.2.1.1	Understand that a relationship between two variables, *x* and *y,* is proportional if it can be expressed in the form $\frac{y}{x}=k$ or $y=kx$. Distinguish proportional relationships from other relationships, including inversely proportional relationships ($xy=k$ or $y=\frac{k}{x}$). *For example*: The radius and circumference of a circle are proportional, whereas the length *x* and the width *y* of a rectangle with area 12 are inversely proportional, since *xy* = 12 or equivalently, $y=\frac{12}{x}$.
			7.2.1.2	Understand that the graph of a proportional relationship is a line through the origin whose slope is the unit rate (constant of proportionality). Know how to use graphing technology to examine what happens to a line when the unit rate is changed.
		Recognize proportional relationships in real-world and mathematical situations; represent these and other relationships with tables, verbal descriptions, symbols and graphs; solve problems involving proportional relationships and explain results in the original context.	7.2.2.1	Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another. Determine the unit rate (constant of proportionality or slope) given any of these representations. *For example*: Larry drives 114 miles and uses 5 gallons of gasoline. Sue drives 300 miles and uses 11.5 gallons of gasoline. Use equations and graphs to compare fuel efficiency and to determine the costs of various trips.
			7.2.2.2	Solve multi-step problems involving proportional relationships in numerous contexts. *For example*: Distance-time, percent increase or decrease, discounts, tips, unit pricing, lengths in similar geometric figures, and unit conversion when a conversion factor is given, including conversion between different measurement systems. *Another example*: How many kilometers are there in 26.2 miles?
			7.2.2.3	Use knowledge of proportions to assess the reasonableness of solutions. *For example*: Recognize that it would be unreasonable for a cashier to request $200 if you purchase a $225 item at 25% off.
			7.2.2.4	Represent real-world or mathematical situations using equations and inequalities involving variables and positive and negative rational numbers. *For example*: "Four-fifths is three greater than the opposite of a number" can be represented as $\frac{4}{5}=-n+3$, and "height no bigger than half the radius" can be represented as $h\le\frac{r}{2}$. *Another example*: "*x* is at least -3 and less than 5" can be represented as $-3\le x<5$, and also on a number line.

7	Algebra	Apply understanding of order of operations and algebraic properties to generate equivalent numerical and algebraic expressions containing positive and negative rational numbers and grouping symbols; evaluate such expressions.	7.2.3.1	Generate equivalent numerical and algebraic expressions containing rational numbers and whole number exponents. Properties of algebra include associative, commutative and distributive laws. *For example*: Combine like terms (use the distributive law) to write $3x-7x+1=(3-7)x+1=-4x+1$.
			7.2.3.2	Evaluate algebraic expressions containing rational numbers and whole number exponents at specified values of their variables. *For example*: Evaluate the expression $\frac{1}{3}(2x-5)^2$ at $x = 5$.
			7.2.3.3	Apply understanding of order of operations and grouping symbols when using calculators and other technologies. *For example*: Recognize the conventions of using a carat (^ raise to a power), asterisk (* multiply), and also pay careful attention to the use of nested parentheses.
		Represent real-world and mathematical situations using equations with variables. Solve equations symbolically, using the properties of equality. Also solve equations graphically and numerically. Interpret solutions in the original context.	7.2.4.1	Represent relationships in various contexts with equations involving variables and positive and negative rational numbers. Use the properties of equality to solve for the value of a variable. Interpret the solution in the original context. *For example*: Solve for w in the equation $P = 2w + 2\ell$ when $P = 3.5$ and $\ell = 0.4$. *Another example*: To post an Internet website, Mary must pay $300 for initial set up and a monthly fee of $12. She has $842 in savings, how long can she sustain her website?
			7.2.4.2	Solve equations resulting from proportional relationships in various contexts. *For example*: Given the side lengths of one triangle and one side length of a second triangle that is similar to the first, find the remaining side lengths of the second triangle. *Another example*: Determine the price of 12 yards of ribbon if 5 yards of ribbon cost $1.85.

	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.1	Understand that a function is a relationship between an independent variable and a dependent variable in which the value of the independent variable determines the value of the dependent variable. Use functional notation, such as $f(x)$, to represent such relationships. *For example*: The relationship between the area of a square and the side length can be expressed as $f(x)=x^2$. In this case, $f(5)=25$, which represents the fact that a square of side length 5 units has area 25 units squared.
			8.2.1.2	Use linear functions to represent relationships in which changing the input variable by some amount leads to a change in the output variable that is a constant times that amount. *For example*: Uncle Jim gave Emily \$50 on the day she was born and \$25 on each birthday after that. The function $f(x)=50+25x$ represents the amount of money Jim has given after x years. The rate of change is \$25 per year.
8	Algebra	Understand the concept of function in real-world and mathematical situations, and distinguish between linear and non-linear functions.	8.2.1.3	Understand that a function is linear if it can be expressed in the form $f(x)=mx+b$ or if its graph is a straight line. *For example*: The function $f(x)=x^2$ is not a linear function because its graph contains the points (1,1), (-1,1) and (0,0), which are not on a straight line.
			8.2.1.4	Understand that an arithmetic sequence is a linear function that can be expressed in the form $f(x)=mx+b$, where $x = 0, 1, 2, 3,\ldots$ *For example*: The arithmetic sequence 3, 7, 11, 15, …, can be expressed as $f(x) = 4x + 3$.
			8.2.1.5	Understand that a geometric sequence is a non-linear function that can be expressed in the form $f(x)=ab^x$, where $x = 0, 1, 2, 3,\ldots$ *For example*: The geometric sequence 6, 12, 24, 48, … , can be expressed in the form $f(x) = 6(2^x)$.

8	Algebra	Recognize linear functions in real-world and mathematical situations; represent linear functions and other functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions and explain results in the original context.	8.2.2.1	Represent linear functions with tables, verbal descriptions, symbols, equations and graphs; translate from one representation to another.
			8.2.2.2	Identify graphical properties of linear functions including slopes and intercepts. Know that the slope equals the rate of change, and that the y-intercept is zero when the function represents a proportional relationship.
			8.2.2.3	Identify how coefficient changes in the equation $f(x) = mx + b$ affect the graphs of linear functions. Know how to use graphing technology to examine these effects.
			8.2.2.4	Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl starts with $100 in savings and adds $10 at the end of each month, she will have $100 + 10x$ dollars after x months.
			8.2.2.5	Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems. *For example*: If a girl invests $100 at 10% annual interest, she will have $100(1.1^x)$ dollars after x years.
8	Algebra	Generate equivalent numerical and algebraic expressions and use algebraic properties to evaluate expressions.	8.2.3.1	Evaluate algebraic expressions, including expressions containing radicals and absolute values, at specified values of their variables. *For example*: Evaluate $\pi r^2 h$ when $r = 3$ and $h = 0.5$, and then use an approximation of π, to obtain an approximate answer.
			8.2.3.2	Justify steps in generating equivalent expressions by identifying the properties used, including the properties of algebra. Properties include the associative, commutative and distributive laws, and the order of operations, including grouping symbols.

8	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.1	Use linear equations to represent situations involving a constant rate of change, including proportional and non-proportional relationships. *For example*: For a cylinder with fixed radius of length 5, the surface area $A = 2\pi(5)h + 2\pi(5)^2 = 10\pi h + 50\pi$, is a linear function of the height h, but it is not proportional to the height.
			8.2.4.2	Solve multi-step equations in one variable. Solve for one variable in a multi-variable equation in terms of the other variables. Justify the steps by identifying the properties of equalities used. *For example*: The equation $10x + 17 = 3x$ can be changed to $7x + 17 = 0$, and then to $7x = -17$ by adding/subtracting the same quantities to both sides. These changes do not change the solution of the equation. *Another example*: Express the radius of a circle in terms of its circumference.
			8.2.4.3	Express linear equations in slope-intercept, point-slope and standard forms, and convert between these forms. Given sufficient information, find an equation of a line. *For example*: Determine an equation of the line through the points (-1,6) and (2/3, -3/4).
			8.2.4.4	Use linear inequalities to represent relationships in various contexts. *For example*: A gas station charges \$0.10 less per gallon of gasoline if a customer also gets a car wash. Without the car wash, gas costs \$2.79 per gallon. The car wash is \$8.95. What are the possible amounts (in gallons) of gasoline that you can buy if you also get a car wash and can spend at most \$35?
			8.2.4.5	Solve linear inequalities using properties of inequalities. Graph the solutions on a number line. *For example*: The inequality $-3x < 6$ is equivalent to $x > -2$, which can be represented on the number line by shading in the interval to the right of -2.
			8.2.4.6	Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities and graph the solutions on a number line. *For example*: A cylindrical machine part is manufactured with a radius of 2.1 cm, with a tolerance of 1/100 cm. The radius r satisfies the inequality $\|r - 2.1\| \leq .01$.

8	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear expressions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	8.2.4.7	Represent relationships in various contexts using systems of linear equations. Solve systems of linear equations in two variables symbolically, graphically and numerically. *For example*: Marty's cell phone company charges $15 per month plus $0.04 per minute for each call. Jeannine's company charges $0.25 per minute. Use a system of equations to determine the advantages of each plan based on the number of minutes used.
			8.2.4.8	Understand that a system of linear equations may have no solution, one solution, or an infinite number of solutions. Relate the number of solutions to pairs of lines that are intersecting, parallel or identical. Check whether a pair of numbers satisfies a system of two linear equations in two unknowns by substituting the numbers into both equations.
			8.2.4.9	Use the relationship between square roots and squares of a number to solve problems. *For example*: If $\pi x^2 = 5$, then $\lvert x\rvert = \sqrt{\frac{5}{\pi}}$, or equivalently, $x = \sqrt{\frac{5}{\pi}}$ or $x = -\sqrt{\frac{5}{\pi}}$. If x is understood as the radius of a circle in this example, then the negative solution should be discarded and $x = \sqrt{\frac{5}{\pi}}$.

9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods.	9.2.1.1	Understand the definition of a function. Use functional notation and evaluate a function at a given point in its domain. *For example*: If $f(x) = \frac{1}{x^2 - 3}$, find $f(-4)$.
			9.2.1.2	Distinguish between functions and other relations defined symbolically, graphically or in tabular form.
			9.2.1.3	Find the domain of a function defined symbolically, graphically or in a real-world context. *For example*: The formula $f(x) = \pi x^2$ can represent a function whose domain is all real numbers, but in the context of the area of a circle, the domain would be restricted to positive x.
			9.2.1.4	Obtain information and draw conclusions from graphs of functions and other relations. *For example*: If a graph shows the relationship between the elapsed flight time of a golf ball at a given moment and its height at that same moment, identify the time interval during which the ball is at least 100 feet above the ground.

9, 10, 11	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.	9.2.1.5	Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - \mathrm{h})^2 + k$, or in factored form.
			9.2.1.6	Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.
			9.2.1.7	Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
			9.2.1.8	Make qualitative statements about the rate of change of a function, based on its graph or table of values. *For example*: The function $f(x) = 3^x$ increases for all x, but it increases faster when $x > 2$ than it does when $x < 2$.
			9.2.1.9	Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. *For example*: Determine how the graph of $f(x) = \lvert x - h \rvert + k$ changes as h and k change.
		Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.1	Represent and solve problems in various contexts using linear and quadratic functions. *For example*: Write a function that represents the area of a rectangular garden that can be surrounded with 32 feet of fencing, and use the function to determine the possible dimensions of such a garden if the area must be at least 50 square feet.
			9.2.2.2	Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth.
			9.2.2.3	Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.
			9.2.2.4	Express the terms in a geometric sequence recursively and by giving an explicit (closed form) formula, and express the partial sums of a geometric series recursively. *For example*: A closed form formula for the terms t_n in the geometric sequence 3, 6, 12, 24, ... is $t_n = 3(2)^{n-1}$, where $n = 1, 2, 3, \ldots$, and this sequence can be expressed recursively by writing $t_1 = 3$ and $t_n = 2t_{n-1}$, for $n \geq 2$. *Another example*: the partial sums s_n of the series $3 + 6 + 12 + 24 + \ldots$ can be expressed recursively by writing $s_1 = 3$ and $s_n = 3 + 2s_{n-1}$, for $n \geq 2$.

		Recognize linear, quadratic, exponential and other common functions in real-world and mathematical situations; represent these functions with tables, verbal descriptions, symbols and graphs; solve problems involving these functions, and explain results in the original context.	9.2.2.5	Recognize and solve problems that can be modeled using finite geometric sequences and series, such as home mortgage and other compound interest examples. Know how to use spreadsheets and calculators to explore geometric sequences and series in various contexts.
			9.2.2.6	Sketch the graphs of common non-linear functions such as $f(x)=\sqrt{x}$, $f(x)=\lvert x\rvert$, $f(x)=\frac{1}{x}$, $f(x)=x^3$, and translations of these functions, such as $f(x)=\sqrt{x-2}+4$. Know how to use graphing technology to graph these functions.
9, 10, 11	Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.1	Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified points in their domains.
			9.2.3.2	Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
			9.2.3.3	Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares. *For example*: $9x^6 - x^4 = (3x^3 - x^2)(3x^3 + x^2)$.
			9.2.3.4	Add, subtract, multiply, divide and simplify algebraic fractions. *For example*: $\frac{1}{1-x}+\frac{x}{1+x}$ is equivalent to $\frac{1+2x-x^2}{1-x^2}$.
			9.2.3.5	Check whether a given complex number is a solution of a quadratic equation by substituting it for the variable and evaluating the expression, using arithmetic with complex numbers. *For example*: The complex number $\frac{1+i}{2}$ is a solution of $2x^2 - 2x + 1 = 0$, since $2\left(\frac{1+i}{2}\right)^2-2\left(\frac{1+i}{2}\right)+1=i-(1+i)+1=0$.

9, 10, 11	Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.6	Apply the properties of positive and negative rational exponents to generate equivalent algebraic expressions, including those involving n^{th} roots. *For example*: $\sqrt{2}\times\sqrt{7}=2^{\frac{1}{2}}\times 7^{\frac{1}{2}}=14^{\frac{1}{2}}=\sqrt{14}$. Rules for computing directly with radicals may also be used: $\sqrt{2}\times\sqrt{x}=\sqrt{2x}$.
			9.2.3.7	Justify steps in generating equivalent expressions by identifying the properties used. Use substitution to check the equality of expressions for some particular values of the variables; recognize that checking with substitution does not guarantee equality of expressions for all values of the variables.
		Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential, and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.1	Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities. *For example*: A diver jumps from a 20 meter platform with an upward velocity of 3 meters per second. In finding the time at which the diver hits the surface of the water, the resulting quadratic equation has a positive and a negative solution. The negative solution should be discarded because of the context.
			9.2.4.2	Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically. Know how to use calculators, graphing utilities or other technology to solve these equations.
			9.2.4.3	Recognize that to solve certain equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers, and from real numbers to complex numbers. In particular, non-real complex numbers are needed to solve some quadratic equations with real coefficients.
			9.2.4.4	Represent relationships in various contexts using systems of linear inequalities; solve them graphically. Indicate which parts of the boundary are included in and excluded from the solution set using solid and dotted lines.
			9.2.4.5	Solve linear programming problems in two variables using graphical methods.

9, 10, 11	Algebra	Represent real-world and mathematical situations using equations and inequalities involving linear, quadratic, exponential and nth root functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.	9.2.4.6	Represent relationships in various contexts using absolute value inequalities in two variables; solve them graphically. *For example*: If a pipe is to be cut to a length of 5 meters accurate to within a tenth of its diameter, the relationship between the length x of the pipe and its diameter y satisfies the inequality $\lvert x - 5\rvert \leq 0.1y$.
			9.2.4.7	Solve equations that contain radical expressions. Recognize that extraneous solutions may arise when using symbolic methods. *For example*: The equation $\sqrt{x-9} = 9\sqrt{x}$ may be solved by squaring both sides to obtain $x - 9 = 81x$, which has the solution $x = -\frac{9}{80}$. However, this is not a solution of the original equation, so it is an extraneous solution that should be discarded. The original equation has no solution in this case. *Another example*: Solve $\sqrt[3]{-x+1} = -5$.
			9.2.4.8	Assess the reasonableness of a solution in its given context and compare the solution to appropriate graphical or numerical estimates; interpret a solution in the original context.