Math 226B, Final Exam, Takehome Part, 13 May 2009

Notes: This test is due at **9:00 pm, Monday, May 18** (at the conclusion of the in-class part of the final). The test is open book and open notes. Don't use any other sources. It's OK to use a calculator, but turn in neat written work that shows very clearly what you did.

Do Problems 1 and 2, and any three of the remaining five problems.

Turn in this page with your solutions; don't forget to consider the pledge.

1. (12 pts) Throughout this problem, let $\langle P, Q\rangle = \langle xy - 2y^2, x + y\rangle$ and let R be the region (in the xy-plane) bounded below by the x-axis and above by the unit circle $x^2 + y^2 = 1$. Let C be the boundary of this region, directed counterclockwise. (Note that C has two parts: a piece of the x-axis and a piece of the unit circle. (You might call these parts C_1 and C_2.)

   (a) Calculate $\displaystyle\int_C P\,dx + Q\,dy$ by parametrizing C_1 and C_2.

   (b) Calculate $\displaystyle\iint_R (Q_x - P_y)\,dA$. (According to Green's theorem, you should get the same answer as in (a).

   (c) Is the Fundamental Theorem for Line Integrals (page 1082) any use in calculating $\displaystyle\int_C P\,dx + Q\,dy$ yet again? If so, do so. If not, explain why not.

2. (8 pts)

   (a) Explain briefly why $\displaystyle\int_C \frac{y}{2}\,dx + \frac{x}{2}\,dy = 0$ for every *closed* smooth curve C in the xy-plane.

   (b) Explain briefly why $\displaystyle\int_C 3\,dx + 5\,dy$ has the same value for *every* smooth curve C running from $(0, 0)$ to (π, e). Find this value.

3. (10 pts) In each part, use the Fundamental Theorem for Line Integrals to calculate the integral $\displaystyle\int_C P\,dx + Q\,dy$.

   (a) $\langle P, Q\rangle = \langle 1 + y\cos(xy), x\cos(xy)\rangle$; C is the straight-line segment from $(0, 0)$ to $(\pi, 1/2)$.

   (b) $\langle P, Q\rangle = \langle 3x + y, x - y^2\rangle$; C is the straight-line segment from $(0, 0)$ to $(3, 2)$.

4. (10 pts) Throughout this problem, let I be the integral $\displaystyle\iint_R (3x^2 + 2y)\,dA$, where $R = [0, 3] \times [0, 3]$.

   (a) Find, showing all work by hand, a double midpoint sum with with a total of 9 subdivisions (i.e., use a 3-by-3 grid). State the answer as a fraction (e.g., 22/3), not in rounded form (e.g., 7.667).

   (b) Calculate I exactly as an iterated integral.

5. (10 pts) Calculate $I = \displaystyle\int_{-1}^{1}\int_{x=y^2}^{x=y+2} (x + y)\,dx\,dy$. Then reverse the order of integration and find I again. (In other words, find I again by calculating one or more integrals that end in $dy\,dx$ rather than $dx\,dy$.) Draw pictures as necessary to illustrate your reasoning.

6. (10 pts) Do #32a and #40, pages 1080 and 1081.

7. (10 pts) Do #20, page 1056

Optional, for 3 pts extra credit. Do #4, page 1096.

Pledge: I pledge my honor that I have neither seen nor done dishonest work on this test.

Signed: .....

I have intentionally not signed the pledge (initials) .....