Solution, Problem 64, page 839

Problem. We're given the line ℓ_0 with parametric equations $x = 1 + t$, $y = 1 - t$, $z = 2t$. Note that this line has direction vector $\vec{v_0} =< 1, -1, 2 >$. The problem is to find an equation for the line ℓ_1 through the point $P_1(0, 1, 2)$ that is both perpendicular to ℓ_0 and hits ℓ_0 somewhere.

Solution. There are various more or less equivalent approaches, including some I mentioned, rather vaguely, in class. The easiest approach (I think) is to find a point P_0 on the original line ℓ_0 so that the vector joining P_0 to P_1 is perpendicular to the line ℓ_0—i.e., to the vector $\vec{v_0} =< 1, -1, 2 >$.

Well, any point P_0 on the line $\ell + 0$ has the form $P_0 = (1 + t, 1 - t, 2t)$, so the vector from P_0 to P_1 has the form

$$P_1 - P_0 =< 0 - (1 + t), 1 - (1 - t), 2 - 2t) >=< -1 - t, t, 2 - 2t >,$$

and this is perpendicular to $\vec{v_0} =< 1, -1, 2 >$ if and only if

$$0 =< -1 - t, t, 2 - 2t > \cdot < 1, -1, 2 >= 3 - 6t.$$

This happens if and only if $t = 1/2$, and so the point we're looking for on ℓ_0 is $P_0 = (3/2, 1/2, 1)$.

For this t, we get

$$P_1 - P_0 =< -1 - t, t, 2 - 2t >=< -3/2, 1/2, 1 >,$$

and we can use this as the direction vector for the desired line ℓ_1.

So the desired line can be written in parametric form as

$$x = 0 - 1.5t; \qquad y = 1 + 0.5t; \qquad z = 2 + t.$$