

## Proof Mechanics – Math 244 – Spring 2007

**1. Negations** Following are some statements. Write the *negation* of each one. Then decide (if you can ... one is quite hard) whether each statement is true or false. Don't prove anything.

P: All dogs have fleas.

Q: Some dogs have fleas.

R: My dog, Spike, is flea-free.

S:  $\sqrt{n-1} + \sqrt{n+1}$  is irrational for every positive integer  $n$ .

T:  $\sqrt{n-1} + \sqrt{n+1}$  is rational for every positive integer  $n$ .

U:  $\sqrt{n-1} + \sqrt{n+1}$  is rational for some positive integer  $n$ .

V:  $\cos(x) < 1.001$  for all  $x > 0$ .

W:  $\cos(x) > 0.999$  for all  $x > 0$ .

X: Every positive even integer  $n \geq 6$  is the sum of two odd primes.

Y: Every subset of the interval  $[0, 1]$  contains a largest element.

Z: Every set of 1234 distinct numbers has a largest and a smallest element.

**2. Counterexamples** At least three of the statements in 1. are false as written. Choose any three of these false statements. For each one, carefully state a *counterexample* that shows the statement to be false.<sup>1</sup>

**3. Converses and contrapositives** In each part following is an implication (i.e., a statement of the form "If P then Q"). In each part, first write the converse and the contrapositive of the given implication. Then label each of the three statements true or false; no need to prove anything.

(a) If  $x > 3$  then  $x^2 > 9$ .

(b) If  $x > 3$  then  $x^3 - 4x^2 + 3x > 0$ .

(c) If  $a > 0$  and  $b > 0$  then  $|a + b| = |a| + |b|$ .

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<sup>1</sup>If you successfully find a counterexample to statement **X** you will receive a free automobile!