Calculus from Graphical, Numerical, and Symbolic Points of View — Overview of 2nd Edition

General notes. These informal notes briefly overview plans for the 2nd edition (2/e) of the Ostebee/Zorn text. This document should be read together with the 2/e Table of Contents, which lists individual sections.

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Chapter 1: Functions and Derivatives: The Graphical View

Overview: We move quickly to the subject of derivatives and rate functions, giving the graphical viewpoint first. Standard elementary functions appear as examples, but we don't—in this chapter—use non-trivial symbolic properties of these functions. The Field Guide section briefly introduces the standard elementary functions, but concentrates (for the moment) mainly on their graphical properties.

Changes from 1/e: We've reduced the time spent in getting to the "real" calculus. Some "precalculus" material has been moved to appendices (the former section on Machine Graphics is one example). We've also consolidated and streamlined coverage of some topics. We hope that the new arrangement will, by moving more quickly to symbolic viewpoints, allow students' problems with algebra to be diagnosed earlier.

Chapter 2: Functions and Derivatives: The Symbolic View

Overview: Here we review and expand on ideas of Chapter 1, but now in the symbolic setting. The ideas of antiderivative and differential equation appear for the first time—systematic treatments appear later. Sections 5, 6, and 7 describe some classical applications (linear motion, growth, and oscillation) in connection with finding derivatives and (in the simplest cases) antiderivatives of polynomial, exponential, and trigonometric functions. Doing so also adds a little "modeling" flavor to the package.

Changes from 1/e: Chapter 2 in 2/e covers material through basic derivatives of exponential and trigonometric functions—these topics appeared in Chapter 3 of the first edition. DEs also appear a little earlier and more often than in the first edition. Mentioning the DE idea early lets us say, for instance, that exponential functions are important largely because they model an important type of growth. *Note.* We do not aim to merge a DE course with the already tight calculus syllabus. We aim only to discuss the *idea* of a DE and its solutions, and to suggest how to use the DE formalism in modeling settings.

Chapter 3: New Derivatives from Old

Overview: This chapter is on "combinatorial" ideas: producing new functions and new derivatives from old functions and their derivatives. We also cover implicit differentiation and derivatives of inverse functions; these ideas are applied to differentiate inverse trigonometric functions.

Changes from 1/e: In 1/e a section at the end of Chapter 1 was about creating new functions from old; we drew on this section in Chapter 3. In 2/e the new-functions-from-old idea is distributed among sections 3.1, 3.2, and 3.4—just in time to be used in calculating derivatives. A new brief section (3.5) collects miscellaneous derivative and (simple) antiderivative problems from earlier sections. Note that systematic antidifferentiation is *not* treated here (it appears in Chapter 8). Here, antidifferentiation is thought of only as differentiation "in reverse."

Chapter 4: Using the Derivative

Overview: A selection of applications, extensions, and uses of the derivative and other ideas from earlier chapters. Some sections are independent of each other; not all are required for later work. Sections 4.8 and 4.9 (on continuity and differentiability) offer glimpses at some of the theoretical side of the subject. In particular, the mean value theorem is discussed and proved fairly carefully in Section 4.10.

Changes from 1/e: Slope fields (aka direction fields) now appear in Section 4.1; this topic appeared in Chapter 12 in 1/e. Section 4.2 offers a revised treatment of limits at infinity and other limit-related ideas—since the derivative is now available, we use it to find and investigate limits. In particular, l'Hôpital's rule now appears in Section 4.2; in 1/e it appeared in Chapter 10. Remaining sections of Chapter 4 all appeared in 1/e, but all have been rewritten and updated, often with additional examples. Sections on splines and economic applications that appeared in 1/e have been omitted, though some of their contents have been redistributed as exercises and projects.

Chapter 5: The Integral

Overview: The first few sections briefly introduce the definite integral geometrically, as signdd area. Then the integral is linked to antiderivatives via the fundamental theorem. Substitution and use of tables are the most basic methods of finding antiderivatives (about which much more is in Chapter 8). The chapter ends with the limit definition of integral. The general treatment of integrals mirrors that of derivatives: we start with geometric intuition and proceed to the limit-based definition.

Changes from 1/e: The new Chapter 5 essentially merges Chapters 5 and 6 from 1/e, to present a basic but unified view of the integral from *both* graphical and symbolic points of view. A new section (5.7) gives students additional help and practice with approximating sums and their uses in estimation. Some instructors might end a one-semester course at this point, having given a basic but comprehensive introduction to the integral idea.

Chapter 6: Numerical Integration

Overview. This chapter treats basic numerical views of the integral; the first section presents and compares different types of approximating sums. A section on Euler's method applies similar numerical ideas to solving DEs. Some error-bound analysis appears in the last section (which some instructors may omit).

Changes from 1/e: This chapter is a completely redesigned version of Chapter 7 of 1/e. Many instructors preferred to defer discussion of error bounds until late in the chapter—or even to omit the subject entirely. The new configuration permits this. Section 6.2, on Euler's method, formerly appeared in slightly different form in Chapter 12. Treatment of Simpson's rule (formerly its own section) is now done as a project.

Chapter 7: Using the Integral

Overview. Applications, extensions, and uses of the integral: geometric, physical, and probabilistic. As in Chapter 4, some sections are independent of each other. The first section offers a variety of applications, including arclength, chosen partly to show the integral as a tool for measuring things—and the fact that the things measured need *not* be interpreted as area. Some integrals that arise might be done with the help of numerical methods, tables, or computers.

Changes from 1/e: One new section (7.4) treats basic continuous probability applications. Another (7.5) applies symbolic integration to solve separable DEs—in 1/e, the latter section appeared in Chapter 12. A section on present value (it was a section in 1/e) will now appear as an (optional) project.

Chapter 8: Symbolic Antidifferentiation Techniques

Overview. The chapter contains a fairly standard menu of techniques. Even though antidifferentiation is readily handled by computer nowadays, we think that some practice of this sort is useful for building symbol-manipulation skills of the sort many students will need in later courses; it also requires students to recognize and grapple with the structure of various classes of functions. "Applications" from the preceding chapter add interest and verisimilitude to some of the exercises.

Changes from 1/e: Few important changes are planned from the treatment in 1/e. However, we do plan a project on the general subject of functions defined by integrals (such as the erf and Fresnel functions).

Chapter 9: Function Approximation

Overview. The chapter itself is entirely new to 2/e, though most of its contents appeared elsewhere in 1/e. The general theme of approximating one function with another appears from the start (e.g., in connection with differentiable functions being "almost" linear functions). Sections 9.1 and 9.2, on Taylor polynomials and Taylor's theorem, are intended to help distentangle these important ideas from issues of series convergence and divergence. (These quite different ideas are too often conflated.) This configuration will let teachers who wish to do so cover Taylor's theorem without also requiring a long introduction to infinite series.

Changes from 1/e: The whole chapter is new; see above.

Chapter 10: Improper Integrals

Overview: Improper integrals are both practically useful and closely analogous to infinite series—hence our relatively extended treatment. Many texts cover the subject in only one section, but we hope to use improper integrals to build intuition for the difficult ideas of infinite series, coming in Chapter 11.

Changes from 1/e: The 2/e version is a subset of the 1/e version, which included sections on l'Hôpital's rule and on integral-based probability. (These topics now appear in earlier sections, though they could be delayed until this point.)

Chapter 11: Infinite Series

Overview: Unlike some reform texts, we do treat convergence and divergence of numerical series, but in a somewhat non-traditional way. We stress (i) the analogy with improper integrals; (ii) concrete, sometimes graphical treatment of partial sums; and (iii) numerical estimation of limits. We think these strategies help make this difficult subject more concrete and accessible than it often is when the principal concern is the abstract question of convergence or divergence. The chapter ends with power series—but with the new Chapter 9 students will already have some exposure to this idea.

Changes from 1/e: Treatment does not differ radically from 1/e version, but earlier exposure to Taylor polynomials (see overview above) should ease passage through traditionally difficult topics.

Chapter V: Vectors and Polar Coordinates

Overview: A basic introduction to 2d-vectors (vectors in the plane; vector notation; vector-valued functions and derivatives) and to the polar coordinate view of the plane and of plane curves. *Note.* Students who continue to Calculus III will see this material in more detail in Chapter 12—this treatment is designed mainly for students who will not continue in calculus.

Changes from 1/e: The polar coordinate material appeared in Chapter 13 of 1/e; the brief treatment of vectors is new.

Chapter M: Multivariable Calculus: A First Look

Overview: A basic introduction to rudiments of functions of several variables. Some instructors may choose to cover it as an alternative to (or, if time permits) in addition to infinite series. Some instructors might choose material on partial derivatives but not on multiple integrals.

Changes from 1/e: Only minor changes are contemplated (e.g., additional exercises). *Note.* Students who continue to Calculus III will see this material in more detail scattered through Chapters 13 and 14—this treatment is designed mainly for students who will not continue in calculus.

Chapter 12: Curves and Vectors

Overview: Introduces vectors and their properties, first in a relatively simple context: curves (mostly defined parametrically) in the xy-plane. Explicit parametrization of curves is stressed, as is idea that curves can be parametrized in various ways, but that some properties of curves don't depend on parametrization.

Changes from 1/e: Changes in narrative will be minor; exercises will be added.

Chapter 13: Derivatives

Overview: The idea of derivative has various incarnations in several variables. We survey them here, together with some standard applications. An important theme is to generalize from intuition about one-variable situations to higher-dimensional settings. Another key theme is "linear approximation": differentiable functions of several variables are close to linear, so care with understanding linear functions is well-rewarded.

Changes from 1/e: Changes in narrative will be minor; exercises will be added.

Chapter 14: Integrals

Overview: The idea, meaning, and some applications of multiple integrals, in various coordinate systems. The general change-of-variable formula, at the end, unifies several earlier ideas.

Changes from 1/e: Only minor changes in narrative; exercises will be added.

Chapter 15: Other Topics

Overview: A sampler of extensions and applications of ideas developed earlier; some are presented as student projects. All material here is independent of the sequel. Motion and curve design are two important areas of application.

Changes from 1/e: Some topics are presented in project form; one project explores cycloids and epicycloids; another explores pedal curves.

Chapter 16: Vector Calculus

Overview: An introduction to the basic objects (especially line and surface integrals) and theorems of vector calculus, with emphasis on Green's theorem—the most accessible vector form of the general fundamental theorem of calculus. More general vector calculus theorems (Stokes's theorem and the divergence theorem) are also presented. A unifying theme is the link to the basic fundamental theorem of elementary calculus.

Changes from 1/e: Minor changes in narrative; new exercises will be added.