

# Northfield Reads & Counts Tutor Manual

St. Olaf College and Carleton College  
Northfield Public Schools

**Section 7: Tips For Tutoring Mathematics**

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## **Mathematics Knowledge by Grade Level**

The following are the expectations of what a student will learn of mathematics and the mathematics skills he/she will acquire in each grade level.

### **Third Grade**

- ✓ Uses basic operations of addition and subtraction with one and two digit numbers including regrouping, column addition, multiplication, and division facts
- ✓ Compares and rounds numbers
- ✓ Recognizes fractions
- ✓ Recognizes when estimation is appropriate

### **Fourth Grade**

- ✓ Describes, draws and classifies two and three dimensional geometric figures
- ✓ Recognizes, describes, and creates a variety of complex geometric patterns
- ✓ Applies standard and non-standard measurements in diverse everyday situations
- ✓ Collect, organize, and analyze data to formulate and solve problems
- ✓ Develops and reads tables, charts and graphs
- ✓ Records and interprets probability

### **Fifth Grade**

- ✓ Applies mental math and estimation strategies
- ✓ Demonstrates understanding of our numeration system
- ✓ Uses mathematical properties and recognizes multiple uses of numbers encountered in the real world.
- ✓ Identifies missing factors in problems

### **Sixth Grade**

- ✓ Computes, estimates, and represents numbers in fraction, decimal, and whole number forms
- ✓ Solves real world problems using functions and algebraic representations
- ✓ Collects, organizes, analyzes, and interprets real life and relevant data using probability, tables, charts and graphs



## **Your Role as a Mathematics Tutor**

Math can be a particularly frustrating and intimidating subject, and your student may not like it because he/she thinks “he/she is just not good at math.” There are many different ways of learning and teaching math, and any student can succeed in math when he/she finds the ones that work for him/her. As a tutor, you can encourage students, build on their strengths, help them gain confidence in their abilities, and set an example of liking math. The following are some things to keep in mind when tutoring math.

1. Keep a balance between helping students get up to speed and helping them develop a richer understanding.

An essential part of tutoring is to help students to fill in gaps in their knowledge and to set them straight when they have misunderstandings. Students also sometimes need a fresh chance to make meaning out of certain mathematical concepts. Effective tutors are those who balance the remedial purpose of tutoring with the purpose of providing opportunities for the student to understand a subject in new ways.

2. Find out where the student’s understanding is faulty.

Ask questions to determine what it is that your student does not understand.

For example:

“Can you take me through what you did, step by step?”

“Can you show me with different numbers than the ones you just used?”

“How would you explain your method to a student in the grade before yours?”

3. Build on what the student already understands.

A key fact to remember is that students, even very young students, have ways of understanding mathematics that may not look like the ways in which you learned when you were in grades K through 8. They may even have ways that are different from how their teachers or textbooks approached the concepts. The one-on-one aspect of tutoring has the great advantage of allowing what the student does understand to surface and to guide subsequent tutoring. Therefore it is important for the tutor to ask the students to explain their thinking and, for students in the later grades, it is also important to ask them to make convincing arguments for their approaches.

## **Commonly Asked Questions about Mathematics Tutoring**

- How do I get a session started?  
With older students, it is often helpful to ask “how math class has been,” and “what has been difficult.” For younger children, you might ask, “What have you been doing in math class? Can you show me?”
- Should I use a calculator?  
If your student is using a calculator in class, you will likely find it valuable to use in your tutoring. Look for “No calculator” notes to know which homework assignments must be done by hand.

## Basics of Mathematics Instruction in the Northfield Public Schools

### **Addition Method—Partial Sums**

$$\begin{array}{r} 67 \\ + 85 \\ \hline 152 \end{array}$$

1. Think of 67 as  $60 + 7$
2. Think of 85 as  $80 + 5$
3. Add each part of 67 to each part of 85

$$\begin{array}{r} 67 \\ + 85 \\ \hline 60 + 80 \rightarrow 140 \\ 7 + 5 \rightarrow \quad \underline{+ 12} \\ 152 \end{array}$$

### **Subtraction Method—Partial Difference**

$$\begin{array}{r} 618 \\ - 479 \\ \hline 139 \end{array}$$

1. Think of 618 as  $600 + 10 + 8$
2. Think of 479 as  $400 + 70 + 9$
3. Subtract each part of 618 and 479
4. Always subtract the small number from the big number
5. If the big number is on top, put a + sign in front
6. If the small number is on top, put a – sign in front
7. Total up the numbers to get the answer

$$\begin{array}{r} 618 \\ - 479 \\ \hline 600 - 400 \rightarrow + 200 \\ 70 - 10 \rightarrow \quad - 60 \\ 9 - 8 \rightarrow \quad \quad - \underline{1} \\ 139 \end{array}$$

### Multiplication Method—Partial Products

$$\begin{array}{r} 34 \\ * 26 \\ \hline 884 \end{array}$$

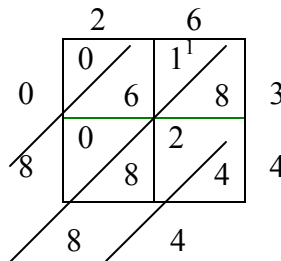
1. Think of 26 as  $20 + 6$
2. Think of 34 as  $30 + 4$
3. Multiply each part of 26 by each part of 34
4. Add the 4 partial products

$$\begin{array}{r} 26 \\ * 34 \\ \hline 30 * 20 \rightarrow 600 \\ 30 * 6 \rightarrow 180 \\ 4 * 20 \rightarrow 80 \\ 4 * 6 \rightarrow 24 \\ \hline 884 \end{array}$$

### Multiplication Method—Lattice

$$\begin{array}{r} 34 \\ \times 26 \\ \hline 884 \end{array}$$

1. Write 26 above the lattice and 34 on the right side of the lattice as shown
2. Multiply  $3 * 6$  and  $3 * 2$ , writing the answer in the lattice as shown  
Multiply by using the “up and out method.”
3. Multiply  $4 * 6$  and  $4 * 2$ , writing the answers in the lattice as shown
4. Add the numbers along each diagonal (moving from right to left) and write the sums just outside the box.
5. When the numbers along the diagonal add up to 10 or more
  - a. record the ones digit
  - b. carry the tens digit to the diagonal above



### Division Method—Partial Quotient

$$371 / 4 = 92 \text{ R}3$$

One Way:

$$\begin{array}{r}
 4 \overline{) 371} \\
 \underline{- 200} \quad | \quad 50 \\
 171 \\
 \underline{- 120} \quad | \quad 30 \\
 51 \\
 \underline{- 40} \quad | \quad 10 \\
 11 \\
 \underline{- 8} \quad | \quad 2 \\
 \hline
 3 \quad | \quad 92
 \end{array}$$

Another way:

$$\begin{array}{r}
 4 \overline{) 371} \\
 \underline{- 200} \quad | \quad 50 \\
 171 \\
 \underline{- 160} \quad | \quad 40 \\
 11 \\
 \underline{- 8} \quad | \quad 2 \\
 \hline
 3 \quad | \quad 92
 \end{array}$$

Another way:

$$\begin{array}{r}
 4 \overline{) 371} \\
 \underline{- 360} \quad | \quad 90 \\
 11 \\
 \underline{- 8} \quad | \quad 2 \\
 \hline
 3 \quad | \quad 92
 \end{array}$$

### Division Method—Traditional

$$371 / 4 = 92 \text{ R}3$$

$$\begin{array}{r}
 92 \text{ R}3 \\
 4 \overline{) 371} \\
 \underline{- 36} \\
 11 \\
 \underline{- 8} \\
 3
 \end{array}$$

Does.....	Divide
McDonald's.....	Multiply
Sell.....	Subtract
Cheese .....	Check
Burgers.....	Bring down

## A Guide for Solving Number Stories

1. Understand what you need to find out.
2. Plan what to do.
3. Estimate the answer.
4. Carry out the plan.
6. Look back to see if your answer agrees with your estimate.

## Order of Operations

1. Do operations inside Parentheses.
2. Calculate all expressions with Exponents.
3. Multiply and Divide in order, from left to right.
4. Add and Subtract in order, from left to right.

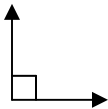
Please.....Parentheses  
Excuse.....Exponents  
My.....Multiplication  
Dear.....Division  
Aunt.....Addition  
Sally.....Subtraction

## Statistical Landmarks

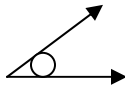
- The minimum is the smallest number.
- The maximum is the largest number.
- The range is the difference between the minimum and the maximum.
- The mode is the number that shows up the most often.
- The median is the exact middle number.
- The mean is the average number of all numbers in a set.

## Classifying Angles

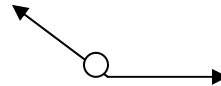
Right Angle =  $90^\circ$



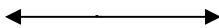
Acute Angle =  $0^\circ$ - $90^\circ$



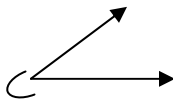
Obtuse Angle =  $90^\circ$ - $180^\circ$



Straight Angle =  $180^\circ$



Reflex Angle =  $180^\circ$ - $360^\circ$



**Line Segments, Rays, Lines and Angles**

**Figure**

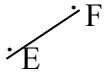
**Name or Symbol**

**Description**

· A

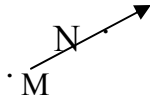
A

Point: a location in space



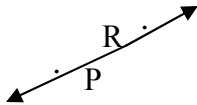
$\overline{EF}$  or  $\overline{FE}$

Line Segment: a straight path between 2 points, called its endpoints



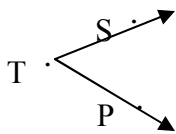
$\overrightarrow{MN}$

Ray: a straight path that goes forever in one direction from an endpoint



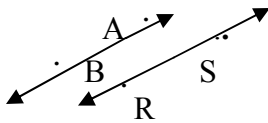
$\leftrightarrow$   $\leftrightarrow$   
PR or RP

Line: a straight path that goes on forever in both directions



$\angle T$  or  $\angle STP$  or  $\angle PTS$

Angle: 2 rays or line segments with a common endpoint called the vertex

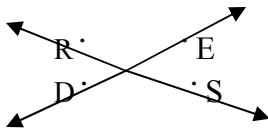


$\leftrightarrow$   $\leftrightarrow$   
AB  $\parallel$  RS

Parallel Lines: Lines that never meet and are everywhere the same distance apart

$\overline{AB} \parallel \overline{RS}$

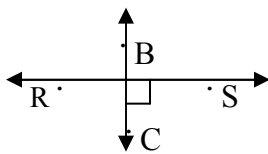
Parallel Line Segments: Segments that are everywhere the same distance apart



none  
none

Intersecting Lines: lines that meet

Intersecting Line Segments: Segments that meet



$\leftrightarrow$   $\leftrightarrow$   
BC  $\perp$  RS

Perpendicular Lines: lines that intersect at right angles

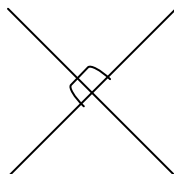
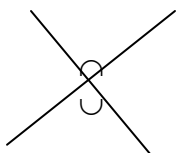
$\overline{BC} \perp \overline{RS}$

Perpendicular Line Segments: Segments that intersect at right angles

**Positions of Angles**

Vertical

Adjacent



## Equivalent Fractions on a Ruler

0 _____	$0/16$ $0/8$ $0/4$ $0/2$
—	$1/16$
—	$2/16$ $1/8$
—	$3/16$
—	$4/16$ $2/8$ $1/4$
—	$5/16$
—	$6/16$ $3/8$
—	$7/16$
—	$8/16$ $4/8$ $2/4$ $1/2$
—	$9/16$
—	$10/16$ $5/8$
—	$11/16$
—	$12/16$ $6/8$ $3/4$
—	$13/16$
—	$14/16$ $7/8$
—	$15/16$
1 _____	$16/16$ $8/8$ $4/4$ $2/2$

## Definitions and Equivalents

<u>Fraction</u>	<u>Mixed Number</u>	<u>Improper Fraction</u>
$\frac{5}{7}$ Numerator	$1 \frac{2}{3}$	$\frac{15}{4}$
7 Denominator	3	4

<u>Fraction</u>	<u>Decimals</u>	<u>Percents</u>
$\frac{3}{4}$	0.75	75%

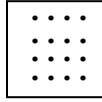
### Stem and leaf plot:

Data: 35, 29, 44, 28, 46, 27, 35, 21, 35, 17, 45

<u>Stems (10s)</u>	<u>Leaves (1s)</u>
1	7
2	1 6 7 8 9
3	5 5 5
4	4 5 6

Square Number (any number that can be written as the product of a number multiplied by itself)

Square array:



Standard Notation

6,000,000,000

Scientific Notation

$6 * 10^9$

Factors

One of two or more numbers that are multiplied to give a product.

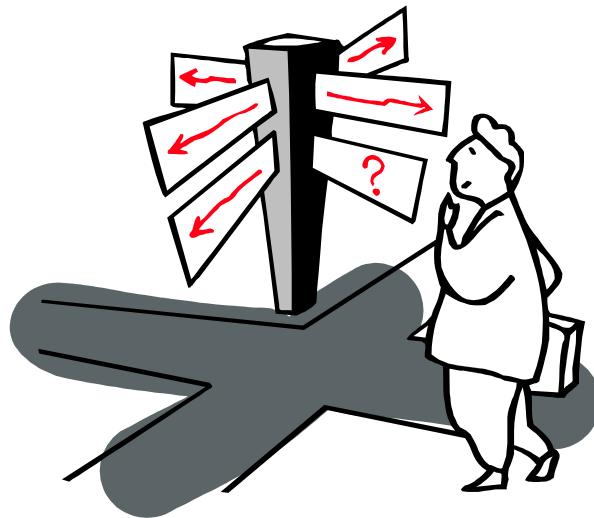
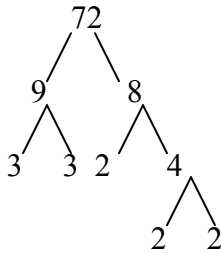
$$9 * 8 = 72$$

$$3 * 3 * 8 = 72$$

Prime Factorization

A whole number expressed as a product of prime factors

$$3 * 3 * 2 * 2 * 2 = 72$$



**Which path helps you the most?**

## **Online Mathematics Resources**

### Math Partners

Great tutoring resources. Go to “download.”

<http://www2.edc.org/mathpartners/>

### America Counts

Great tutoring resources.

<http://www.ed.gov/mericacounts/roadmap/6/index.html>

### Math Ideas

Math activities and games on every subject

<http://www.teachingideas.co.uk/>

### The Math Forum

Go to “Math Resources by Subject”

<http://mathforum.org/>

### PBS

Go to Guide, then Topic, then Lesson Plan Options

<http://www.pbs.org/teachersource/math.htm>

### The Educator’s Reference Desk

Lesson plans by subject

<http://www.eduref.org/Virtual/Lessons/index.shtml>

### National Council of Teachers of Mathematics

Math teaching information and links

<http://www.nctm.org>

### Ask Dr. Math

Extensive records of previously asked and answered math questions divided by age group. You can submit a question but it may or may not get answered.

<http://www.forum.swarthmore.edu/dr.math/dr-math.html>

### For Science Help:

#### Scientific American

An archive of Ask the Experts questions.

<http://www.sciam.com/askexpert>

### For Social Studies Help:

#### Central Intelligence Agency

Comprehensive guide to countries, with maps and encyclopedic information.

<http://www.odci.gov/cia/publications/factbook>