Calculation of basal melt rates.
We employed a 1-D vertical ice compression model to calculate the rate of subglacial melting (and surface accumulation) required to form the internal layer pattern as measured along AA’ (Fig. S1a,b). Internal layers were dated at their intersection with the Byrd ice core. The mean rate of ice accumulation responsible for the burial of an internal layer is given by:

\[ c_{b,t} = \frac{2H - h}{2t} \ln \left( \frac{\frac{3}{2}H - h}{y - h} \right), \quad h \leq y \leq H, \]

where \( c_{b,t} \) is the average accumulation of ice from the present day \((t = 0)\) to time \(t\) (positive as time before present), \(H\) is the total ice thickness, and \(y\) is the distance above the bed of an internal layer. This model requires a value for \(h\), the thickness of the basal shear layer, which has previously been estimated at 400 m in Greenland (S1), although it is acknowledged that the real value is actually not known. Despite this problem, the model is thought to be accurate in the upper ~80% of the ice column (S2).

Determination of fold geometry along AA’
The fold along AA’ was measured in terms of its amplitude and orientation. Amplitude measurements were taken as the difference between the highest elevation of an internal layer and the corresponding lowest elevation at the base of the distortion’s trough both upstream and downstream (Fig. S1c). Orientation (Fig. S1d) was measured by simply plotting datapoints shown in blue in Fig. S1a, without vertical exaggeration. In each case the deepest data point (either in the trough or the crest) is given a horizontal position at zero (Fig. S1d).

Change in fold amplitude and axis along XY
The manner in which the englacial fold changes along XY is a function of both the past flow (which we contend acted to establish the fold) and present flow (which we believe acts to translate and deform the fold). The amplitude of the fold along XY is illustrated in Fig. S1e (fold amplitude versus depth) and Fig. S2a (the separation of the internal layers along the paleoflow line). Both figures show the fold to increase in size from X to Y (although for the first 30 km there is a slight decrease in the fold’s depth-related amplitude; Fig. S1e). The angle of the fold axis (as measured from the RES transect) increases (from the vertical) generally from X to Y, as noted in Fig. S2a (although at Y the angle is slightly less than it is upstream).

Calculation of internal layer submergence along XY
Internal layers can be traced very well in the RES data, allowing the depth of internal layers along the englacial fold axis XY to be determined (Fig. S2a). This longitudinal section shows the steady convergence of internal layers to the bed from X to Y (the basal internal layer converges with the bed by ~600 m). We use a simple 2-dimensional ice flow model (see caption of Fig. S2 and Table S1 for details of the model), orientated along the line of ice flow to determine the conditions that generate the required submergence of internal layers (Fig. S2b-g). In the first three experiments, ice flow across a no-sliding/sliding transition was modeled (S3). When a basal slip ratio of \(c = 1\) is used, the positions of internal layers are displaced downwards by around 300 m (Figs. S2a and S2b). Internal layers were able to be displaced downwards by around 400 m using a basal slip ratio of \(c = \)}
Increasing the slip ratio to $c = 100$ and $c = 500$ resulted in slightly more ice sheet ‘downdraw’, but with an associated significant modification to the ice sheet profile (Fig. S2d,e). Only when subglacial melting was accounted for ($m = 5 \text{ cm yr}^{-1}$ and $m = 10 \text{ cm yr}^{-1}$; Fig. S2f,g), under a slip ratio of $c = 5$, were the internal layers displaced downwards across the melt/onset zone by in excess of 500 m across a 50 km section of the transect without significant change to the ice sheet profile. The model does not account for the effect of lateral convergence of ice into the englacial trough, although if downdraw takes place this is likely to occur. This omission does not adversely affect the results, however, as we assume that the ice flowline at the centre of the fold is along its axis. We are, thus, able to explain the internal layer structures seen in Figures 1c and 2 in terms of ice sheet downdraw caused by the transition from no sliding to sliding in conjunction with basal melting, at a time when the ice flow direction was about $30^\circ$ to the current direction at Y.
Figure S1. (a) RES data along transect AA’ (see Fig. 1 and (f) for location). (b) Rates of surface ice accumulation (blue line) and subglacial melting (red line) along AA’. (c) Amplitude of the englacial fold along AA’, with depth. Datapoints are denoted by blue crosses in (a). (d) Orientation of the fold axis along AA’. In (a), the fold axis is denoted by right hand set of blue cross, with two sets to the left denoting the orientation of the structure 5 and 10 km upstream of the fold axis. (e) Amplitude (versus depth) of the englacial fold along XY. The horizontal distances between crest and trough are noted for four positions, and are located in (f).
Figure S2. (a) Cross section of the ice sheet along XY, constructed from the positions of RES layers and the bed at the intersection of this line with RES transects (all datapoints used to build the cross section are illustrated, and are located in Fig. S1f). The transect denotes the amplitude of the englacial fold trough along XY. Also noted for four locations (Fig. S1f) are the horizontal crest-trough distance, and the angle of the fold axis. (b-g) Numerical ice flow model results of isochrones across a no-sliding/sliding transition zone beneath an ice sheet. A steady-state ice sheet geometry was calculated ($S4$) using the shallow ice approximation to calculate the vertically averaged
horizontal flux along a longitudinal profile, assuming a uniform accumulation rate. The thickness at
the ice sheet front was set to the flotation thickness. The surface geometry, the two-dimensional
horizontal velocity field along the flow and the flowlines (thin black lines) were calculated
analytically. This information was used to calculate internal layers of the same age (isochrones; red
lines) by following the advection of a discrete number of points from the ice surface. The model’s
glaciological setting and the values of its parameters are listed in Table S1. The ice sheet is assumed
to be frozen to the base (no sliding) from the ice divide down to a distance \( x_1 = 300 \) km. From there
a slip ratio \( c \) is assumed, which is defined as the ratio between sliding velocity and the total vertical
deformation of the horizontal velocity over the entire ice thickness. The thick dashed line denotes
the region in which the slip ratio is applied. In addition, between \( x_1 \) and \( x_2 \) a basal melt-rate \( m \) can
be prescribed. (b) Flowlines and isochrones for a slip ratio of \( c = 1 \). (c) Flowlines and isochrones for
a slip ratio of \( c = 5 \). The layers are significantly shifted downwards around the position of the
sliding onset, due to the abrupt change in flow field, especially close to the base. The maximum
downshift of internal (close to the base) is about 400 m. (d) and (e) show that for much higher slip
ratios of \( c = 100 \) and \( c = 500 \), respectively, the relative magnitude of the downshift does not
increase much from that shown in (b). By including subglacial melting in the model with rates of (f)
\( m = 5 \text{ cm yr}^{-1} \) and (g) \( m = 10 \text{ cm yr}^{-1} \), over a distance of 100 km starting at the onset of sliding (\( x = x_1 \), shown as the red part of the thick dashed line across the ice base), enhanced downshifts in the
position of internal layers were generated.

Table S1. Parameters used in the numerical model of ice flow across a no-sliding/sliding transition
(with or without basal melting accounted for).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>( L )</td>
<td>Ice sheet half length (ice divide to grounding line)</td>
<td>1000 km</td>
</tr>
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<td>( b )</td>
<td>Altitude of ice sheet base</td>
<td>-1000 m a.s.l.</td>
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<td>( h_L )</td>
<td>Ice thickness at grounding line</td>
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<td>( A )</td>
<td>Rate factor</td>
<td>( 2.4 \times 10^{-24} \text{ s}^{-1} \text{ Pa}^{-3} )</td>
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<tr>
<td>( a )</td>
<td>Accumulation rate</td>
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<tr>
<td>( n )</td>
<td>Flow law exponent</td>
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<td>( c )</td>
<td>Basal slip ratio</td>
<td>Variable</td>
</tr>
<tr>
<td>( m )</td>
<td>Basal melt rate</td>
<td>Variable</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>Horizontal position of sliding onset (and melting)</td>
<td>300 km</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>Horizontal position of end of subglacial melting</td>
<td>Variable</td>
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References