1. Let $G$ be a cyclic group of order 10, and let $a$ be a generator of $G$. Use the fact that $a^{10} = e$ to find all the elements in the following subgroups.

(a) $\langle a^2 \rangle$
(b) $\langle a^3 \rangle$
(c) $\langle a^4 \rangle$
(d) $\langle a^5 \rangle$

2. **Conjecture:** Let $G$ be a cyclic group of order $n$ and let $a$ be a generator of $G$. The order of the subgroup $\langle a^m \rangle$ is . . . .

3. **Corollary:** $a^m$ is a generator of $G$ if and only if . . . .

4. List all the possible generators of the groups $G$ and $G'$ from problems 1 and 2 above.

5. Compare your answers on this worksheet to the corresponding questions on worksheet 9. Do you see the similarities?

6. Assuming your conjectures from above are correct, answer the following questions.

(a) List all the elements that can generate a cyclic group of order 14 generated by the element $b$.
(b) If $\langle \alpha \beta \rangle$ has prime order $p$, how many different generators does $\langle \alpha \beta \rangle$ have?
(c) Let $v$ be an element of order 21. How many elements are in the subgroup $\langle v^3 \rangle$?
(d) Find two elements in $\langle v \rangle$ that generate a cyclic subgroup of order 3.
7. Consider \( \langle 4 \rangle \subset \mathbb{Z}_{10} \). Since \( \langle 4 \rangle \) is
a cyclic group of order 5, it must be isomorphic to \( \mathbb{Z}_5 \).
Show this is true by (a) explicitly defining an isomorphism
between the two groups, and (b) proving that the map you defined
really does satisfy the three isomorphism properties.

8. **Extra Credit:** Let \( m \in \mathbb{Z}_n \). Prove that
\( \langle m \rangle \cong \mathbb{Z}_{n/d} \), where \( d = \gcd(m, n) \).

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**Homework 10**
Due Friday, March 8

- Read all of §1.5 if you haven’t already.
- §1.5/16,18,20,30,32,43-47
- For
  16, 18 and 20 remember that a generator must go to a generator.

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