• **Theorems:** In this worksheet you will compute explicit examples illustrating the 6 following theorems concerning a homomorphism \( \phi : G \rightarrow G' \).

- If \( H < G \) then \( \phi(H) < G' \).
- If \( K' < G' \) then \( \phi^{-1}(K') < G \).
- Range \( \phi < G' \).
- Ker \( \phi < G \).
- If \( \phi(x) = y \) then \( \phi^{-1}(y) = x \text{ Ker } \phi \) (a left coset).
- \( \phi \) is one-to-one if and only if Ker \( \phi = \{e\} \).

• **Computations:** Consider the homomorphism \( f : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12} \) given by \( f(n \mod 18) = 2n \mod 12 \). So, for example, \( f(7) = 2 \).

1. Find \( f(n) \) for all \( n \in \mathbb{Z}_{18} \).

2. Consider \( H = \langle 3 \rangle < \mathbb{Z}_{18} \). Identify the elements in \( f(H) \). For what \( a \in \mathbb{Z}_{12} \) is \( f(H) = \langle a \rangle \)?

3. Consider \( K' = \langle 4 \rangle < \mathbb{Z}_{12} \). Identify the elements in \( f^{-1}(K') \). For what \( b \in \mathbb{Z}_{18} \) is \( f^{-1}(K') = \langle b \rangle \)?

4. Identify the elements in Range \( f \). For what \( a \in \mathbb{Z}_{12} \) is Range \( f = \langle a \rangle \)?

5. Identify the elements in Ker \( f \). For what \( b \in \mathbb{Z}_{18} \) is Ker \( f = \langle b \rangle \)?

6. Note that \( f(5) = 10 \). Identify the elements in \( f^{-1}(10) \).

7. How many distinct left cosets of Ker \( f \) in \( \mathbb{Z}_{18} \) are there? What are they?
8. What do you notice about your answer to (6) compared with your answer to (7)?

9. Question (6) indicates that $f$ is not one-to-one. Rather it is ____-to-one. Notice also that $|\text{Ker } f| = ____$.

- Learn the proof of LaGrange’s Theorem
- Homework 18: §3.1/ 1,2,3,6,8,9,16,17,18,20,28,29
- Due: Wednesday April 10