Humke’s Almost All Abelian Groups
Work Sheet 22

1. Find all non-isomorphic abelian groups of orders 12, 72, 105, 60025.

2. Conjecture the condition(s) under which there is only one distinct abelian group of order \( n \).

3. Use the fact that \( \mathbb{Z}_n \times \mathbb{Z}_m \cong \mathbb{Z}_{nm} \) if and only if \( \gcd(n, m) = 1 \) to rewrite the following products. You may either “collapse” the products or “expand” them. For example, we can collapse \( \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4 \) to \( \mathbb{Z}_6 \times \mathbb{Z}_4 \). There may be several ways to do this, you should find at least two per problem.
   
   (a) \( \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{10} \)
   
   (b) \( \mathbb{Z}_4 \times \mathbb{Z}_6 \times \mathbb{Z}_{10} \)
   
   (c) \( \mathbb{Z}_9 \times \mathbb{Z}_{50} \)

4. The set \( G = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\} \) is a group under multiplication mod 45.
   
   (a) Find the orders of all the elements in \( G \).
   
   (b) Of the groups you listed above of order 12, which one is isomorphic to \( G \)? (To answer this, compare orders of elements in the 3 groups of order \( \mathbb{Z}_{12} \) to orders of elements in \( G \).)

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**Homework 22**

**Due:** Friday April 19

- §2.4/21,24,43
- Identify the group \( U_{50} \) as a product of cyclic groups
- Show that \( \{(g, g) \mid g \in G\} \) is a subgroup of \( G \times G \).
- Let \( H = \{(x, e) \mid x \in G_1\} \). Prove that \( H \) is a subgroup of \( G_1 \times G_2 \). Then show that \( H \cong G_1 \).