1. Prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{Z}^+$. 

2. Determine which of the following relations is an equivalence relation on $\mathbb{Z}$. If a relation is not an equivalence relation, say which of the three properties hold.
   (a) $x \sim y \iff x = y$
   (b) $x \sim y \iff x \leq y$
   (c) $x \sim y \iff x$ and $y$ are divisible by the exact same powers of 2.

3. Prove that the relation on $\mathbb{Z}$ given below is an equivalence relation (when $n$ is a positive integer).
   $$x \sim y \iff x - y \text{ is divisible by } n$$

4. Suppose $n = 4$ in the relation above.
   (a) List the elements in $\mathbb{Z}_4$. What about $\mathbb{Z}_{12}$?
   (b) How many different equivalence classes are there?
   (c) Use the equivalence relation to partition $\mathbb{Z}$.

5. Is the following relation on the set of all humans an equivalence relation?
   $$x \sim y \iff x \text{ and } y \text{ have the same (biological) parents.}$$

6. Let $P_3$ be the set of all polynomials of degree less than or equal to 3. Let $f(x)$ and $g(x)$ be two polynomials in $P_3$, and define a relation on $P_3$ by:
   $$f \sim g \iff f' = g'.$$
   (a) Is the relation an equivalence relation?
   (b) Describe the elements in the sets $x^2$ and $x^2 + 7x$.
   (c) Describe a partition of $P_3$ into distinct equivalence classes.

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**Homework due Monday, February 18**

- Do the problems above.
- Make sure you have read §0.1–§0.3
- §0.2/29–32, 35, 36a
- §0.3/2, 3, 4