1. Consider the Cayley table for a group $G$ given below.

<table>
<thead>
<tr>
<th></th>
<th>$e$</th>
<th>$\sigma$</th>
<th>$\sigma^2$</th>
<th>$\tau$</th>
<th>$\sigma\tau$</th>
<th>$\sigma^2\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$e$</td>
<td>$\sigma$</td>
<td>$\sigma^2$</td>
<td>$\tau$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\sigma$</td>
<td>$\sigma^2$</td>
<td>$e$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>$\sigma^2$</td>
<td>$e$</td>
<td>$\sigma$</td>
<td>$\tau$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\tau$</td>
<td>$\sigma^2\tau$</td>
<td>$e$</td>
<td>$\sigma$</td>
<td>$\tau$</td>
<td>$\sigma\tau$</td>
</tr>
<tr>
<td>$\sigma\tau$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
<td>$e$</td>
<td>$\sigma^2$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
</tr>
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<td>$\sigma^2\tau$</td>
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<td>$\sigma\tau$</td>
<td>$\tau$</td>
<td>$\sigma$</td>
<td>$\sigma\tau$</td>
<td>$\sigma^2\tau$</td>
</tr>
</tbody>
</table>

(a) Fill in the missing entries.
(b) Is $G$ abelian? Explain.
(c) What is $\sigma^{-1}$? What about $(\sigma\tau)^{-1}$?
(d) Show by example that $x * y = y * z$ does not necessarily imply that $x = z$.
(e) Find the smallest positive integer $n$ so that $\sigma^n = e$. Do the same for $\sigma\tau$.
(f) The number $n$ that you found in the two cases above is called the order of an element. Find the order of $\sigma^2$ and the order of $\tau$.
(g) Identify the elements in the set $\{g \in G \mid g^2 = e\}$.
(h) Identify the elements in the set $\{g^2 \mid g \in G\}$.
(i) Identify the elements in the set $A = \{g \in G \mid \sigma g = g \sigma\}$.
(j) Use the table above to help you construct the multiplication table for the set $A$. Do you think this set is a group?
(k) Do the same as above for the subset $\{e, \sigma, \tau\}$. Do you get a group?

2. Refer to worksheet 5, problem 1. Which of those examples defines a group?

3. Do problem 28 on page 64.

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**Math 252–Homework 7**
Due Wednesday, February 27

- Read §1.3
- Do §1.3/2,4,12,13,19
- On a separate page, do §1.3/30,34,35
• For (12) and (13), remember that the determinant of a diagonal matrix is just the product of the entries along the diagonal.

• For (30), first show that $x = x^{-1}$ for all $x \in G$, then use the fact in your proof.