An auto-homeomorphism of a Cantor set with zero derivative everywhere

Krzysztof Chris Ciesielski

We construct a closed bounded subset $\mathfrak{X}$ of $\mathbb{R}$ with no isolated points which admits a differentiable bijection $\mathfrak{f}\colon \mathfrak{X} \to \mathfrak{X}$ such that $\mathfrak{f}'(x) = 0$ for all $x \in \mathfrak{X}$. We also show that any such function admits a restriction $\mathfrak{f} \restriction P$ to an uncountable closed $P \subseteq \mathfrak{X}$ forming a minimal dynamical system. The existence of such a map $\mathfrak{f}$ *seems* to contradict several well know results. The map $\mathfrak{f}$ marks a limit beyond which Banach Fixed-Point Theorem cannot be generalized.