

**A Bit of the History of the Scottish Book,
and the Solution to Problem 157**

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In this work, joint with Paul Humke and Kevin Beanland we review the very interesting history of one problem from the Scottish Book, a collection of problems in a diverse array of subjects including real analysis, functional analysis, group theory, set theory, probability, and others. These problems were recorded by mathematicians in the Polish school of mathematics and their visitors while meeting at the Scottish Café in Lwów. Problems were contributed by Banach, Steinhaus, Ulam, von Neumann, Sobolev, and many others. Some of these problems were accompanied by promised rewards for solutions, which ranged from a bottle of wine or several bottles of beer to a kilo of bacon to lunch at the Dorothy (presumably a café) in Cambridge. The latter was offered as a reward for a correct solution to Problem 157 which asks the following:

If f is approximately continuous on $[0, 1]$, with positive right approximate upper derivative, is f strictly increasing?

This problem was entered in the Scottish Book in 1937 by the English mathematician A. J. Ward. The standard proof for the continuous case led to a paper by Richard O'Malley on approximate maxima and indeed the affirmative solution to 157 follows rather quickly from this result of O'Malley. However, a discussion with O'Malley led to an earlier work Don Ornstein in 1968 and as far as we see, Ornstein's paper contains the first solution to 157. We isolate the main idea underpinning both the O'Malley result and Ornstein's theorem and also discuss an example found in the Ornstein paper. This construction given has some difficulties, and we will provide a fix for this.