Comparison of some families of real functions in sense of porosity

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We consider set $\mathbb{R}^\mathbb{R}$ with uniform convergence metric, i.e:

$$\rho(f,g) := \max\{1, \sup_{x\in\mathbb{R}} |f(x)-g(x)|\} \quad \text{for } f,g \in \mathbb{R}^\mathbb{R}$$

and the following subsets of $\mathbb{R}^\mathbb{R}$: quasi-continuous ($\mathcal{Q}$) and Świątkowski functions ($\acute{\mathcal{S}}$) and strong Świątkowski functions ($\acute{\mathcal{S}}_s$):

- $f \in \mathcal{Q}$ if for all $a < x < b$ and each $\varepsilon > 0$ there exists an open interval $J \subset (a,b)$ such that $\operatorname{diam} f[J \cup \{x\}] < \varepsilon$.
- $f \in \acute{\mathcal{S}}$ if for all $a < b$ with $f(a) \neq f(b)$, there is a y between $f(a)$ and $f(b)$ and an $x \in (a,b) \cap \mathcal{C}(f)$ such that $f(x) = y$.
- $f \in \acute{\mathcal{S}}_s$ if for all $a < b$ with $f(a) \neq f(b)$ and for all y between $f(a)$ and $f(b)$ there is an $x \in (a,b) \cap \mathcal{C}(f)$ such that $f(x) = y$.

For arbitrary matric space (X,d), $x \in M \subset X$, and $r \in \mathbb{R}_+$ one can define:

$$\gamma(x,r,M) = \sup\{t \geq 0 : \exists_{z\in X} B(z,t) \subset B(x,r) \setminus M\} \text{ and}$$

$$p(M,x) = 2 \limsup_{t\to r^+} \frac{\gamma(x,r,M)}{r}.$$

Quantity $p(M,x)$ is called *porosity of M at the point x*. We say that M is *porous* if $p(M,x) > 0$ for all $x \in M$ and that it is *strongly porous* if $p(M,x) = 1$ for all $x \in M$.

We have compared considered sets in terms of porosity and we obtained the following results:

- the family $\acute{\mathcal{S}}_s$ is strongly porous in $(\mathcal{Q}\acute{\mathcal{S}}, \rho)$,
- the family $\mathcal{Q}\acute{\mathcal{S}}$ is strongly porous in $(\acute{\mathcal{S}}, \rho)$,
- the family $\mathcal{Q}\acute{\mathcal{S}}$ is $1/3$ porous in $(\mathcal{Q}, \rho)$,
- the family $\mathcal{Q}\acute{\mathcal{S}}$ is not strongly porous in $(\mathcal{Q}, \rho)$.