Mapping metric spaces onto cubes by nice mappings

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For $s > 0$, let $\mathcal{H}^s$ be an s-dimensional Hausdorff measure and let $\dim_H X$ denote the Hausdorff dimension of a separable metric space X. Fix a metric space X and $s > 0$. Consider the following two properties: (a) $\mathcal{H}^s(X) > 0$, (b) $\dim_H X \geq s$. Clearly, (a) implies (b). There is an intermediate property: for every countable cover $\{X_n\}$ of X there is n such that $\dim_H X_n \geq s$. We characterize this property within a rather wide class of separable metric spaces by means of mappings of X onto cubes.