## AFTER THE BISHOP-PHELPS THEOREM

A mapping between normed spaces, $F: X \rightarrow Y$, is said to be normattaining if there is $x_{0} \in X,\left\|x_{0}\right\|=1$, such that $\sup _{x \in B_{X}}\|F(x)\|=\left\|F\left(x_{0}\right)\right\|$. (Here, $B_{X}$ is the closed unit ball of $X$.) This expository talk has its origins in work by Victor Klee in the 1950's, but our discussion will begin with the following short and elegant 1961 result of Errett Bishop and Robert Phelps.

Theorem: Let $\varepsilon>0$. For any $X$ and any continuous linear form $\varphi \in X^{*}$, there is a norm-attaining element $\psi \in X^{*}$ such that $\|\varphi-\psi\|<\varepsilon$.

Our intention is to describe some of the branches of study that are direct results of this $1-1 / 2$ page paper during the last $50+$ years. Among the topics that we plan to discuss are norm-attaining linear operators and norm-attaining multilinear functions.

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