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SOME RESULTS RELATED WITH THE RIEMANN-LEBESGUE LEMMA I

We obtain some versions of the Riemann-Lebesgue Lemma in the Henstock-Kurzweil (HK) Integral context. In general, this result is not valid on $HK[a,b]$. On the HK space completion ($\widehat{HK([a,b])}$) the following asymptotic behavior holds.
Let $[a,b]$ a compact interval. If φ is a function on $[a,b]$ to $\mathbb{R}$ such that φ' is bounded and $\varphi(s) = o(s)$ when $|s| \to \infty$, then for each $f \in \widehat{HK([a,b])}$

$$\int_a^b \varphi(st)f(t)dt = o(s),\ |s| \to \infty. \tag{1}$$

Because of the functions sine and cosine are such that $\sin(s) = \cos(s) = o(s)$, $s \to \infty$, then the Fourier Coefficients have a similar behavior as the expression (**??**). So with this result we obtained Fourier Series partial sum has an asymptotic behavior as the expression (**??**), for elements that are in the completion.

The Riemann-Lebesgue Lemma is related to the convergence of integrals of products of the form fg_λ, where f and g_λ satisfy certain conditions for the product to be integrable in some sense. In this case, if $f \in \widehat{HK([a,b])}$ and g is a bounded variation function on $\mathbb{R}$, then

$$\lim_{\lambda\to\infty} \int_{-\infty}^{\infty} f(t)g(\lambda t)dt = \lim_{t\to\infty} g_+(t) \int_0^{\infty} f + \lim_{t\to\infty} g_-(t) \int_{-\infty}^{0} f.$$

We can note that $\lim_{t\to\infty} g_+(t)$ and $\lim_{t\to\infty} g_-(t)$ always exist.

If we consider $A_\lambda = \int_a^b f(t)g(\lambda t)$ as new coefficients, the serie generated is

$$\dot{S}(f,t) = \sum_{\lambda=0}^{\infty} A_\lambda g(\lambda t).$$

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We get an analogous result to Cantor-Lebesgue Lemma. It means that, if $\dot{S}(f,t)$ converges, then the coefficients A_λ tend to zero, as λ goes to infinite. We extend this result for a numerate family of bounded variation functions.