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ON VERY NON-LINEAR SUBSETS OF CONTINUOUS FUNCTIONS

In this work we continue the study initiated by Gurariy and Quarta in 2004 on the existence of linear spaces formed, up to the null vector, by continuous functions that attain the maximum only at one point. Inserting a topological flavor to the subject, we prove that results already known for functions defined on certain subsets of $\mathbb{R}$ are actually true for functions on quite general topological spaces. In the line of the original results of Gurariy and Quarta, we prove that, depending on the desired dimension, such subspaces may exist or not.

More precisely, given a topological space D, by $\widehat{C}(D)$ we denote the subset of the linear space $C(D)$ of all real-valued continuous functions on D composed by the functions that attain the maximum exactly once in D.

The main results obtained by Gurariy and Quarta in this direction are the following:

(A) $\widehat{C}[a,b)$ contains, up to the origin, a 2-dimensional linear subspace of $C[a,b)$.

(B) $\widehat{C}(\mathbb{R})$ contains, up to the origin, a 2-dimensional linear subspace of $C(\mathbb{R})$.

(C) There is no 2-dimensional linear subspace of $C[a,b]$ contained in $\widehat{C}[a,b] \cup 0$.

In this work we extend (A) to spaces of functions defined on topological spaces D that can be continuously embedded onto some Euclidean sphere S^n. We also extend (B) to spaces of functions defined on quite general topological spaces D that include $\mathbb{R}$. In the two former cases we prove that $\widehat{C}(D) \cup 0$ contains an $(n+1)$-dimensional subspace. Finally, we extend (C) to spaces

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of functions defined on compact subsets K of $\mathbb{R}^m$. In this case we prove that $\widehat{C}(K) \cup 0$ does not contain an $(m+1)$-dimensional subspace of $C(K)$ for every compact $K \subset \mathbb{R}^m$ but, on the other hand, there are compact sets $K \subset \mathbb{R}^m$ for which $\widehat{C}(K) \cup 0$ contains an m-dimensional subspace of $C(K)$.

This is a joint work with G. Botelho, D. Cariello, D. Pellegrino and J.B. Seoane-Sepúlveda.