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ON FUNCTIONS WITH CONTINUOUS RESTRICTIONS TO VARIOUS SETS

A function $f\colon \mathbb{R}^n \to \mathbb{R}$ is separately continuous, when it has continuous restriction $f \restriction H$ for all hyperplanes H perpendicular to one of the axes. We will briefly discuss a classical work on this class of functions and explore different generalizations of this notion. In one direction, we will examine maps $f\colon \mathbb{R}^n \to \mathbb{R}$ with continuous restrictions to: (a) all k-dimensional flats (i.e., affine subspaces) of $\mathbb{R}^n$; (b) only the flats that are parallel to subspaces generated by coordinate axis. For each of these classes, we examine the structure of the sets of points discontinuities of its members and show that these families of sets are different for different classes. In another direction, we investigate for what classes G of functions $g\colon \mathbb{R} \to \mathbb{R}$ the continuity of $f\colon \mathbb{R}^2 \to \mathbb{R}$ is insured by the continuity of all its restrictions $f \restriction g$ for $g \in G$. We note, that this is the case for G consisting of all continuously differentiable functions, if we allow infinite derivatives; however, the result is false for twice differentiable functions. We also show, that the continuity of f can be insured for G consisting of all translations of a single Baire class one function g; however, this cannot be achieved when g is continuous.

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